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Merab Shalamberidze
Doctor of Technical Sciences, Professor, Faculty of Technological Engineering, Department of Design and Technology, Akaki Tsereteli State University, Kutaisi, Georgia
merab.sh@hotmail.com

Zaza Sokhadze
Doctor of Mathematic, Professor, Faculty of Natural Sciences, Department of Mathematics, Akaki Tsereteli State University, Kutaisi, Georgia
z.soxadze@gmail.com

SECTION 25. Technologies of materials for the light and textile industry

CONSTRUCTING A SHAPE OF ORTHOPEDIC BOOT-TREE PRINT BY MEANS OF THE SOLUTION TO DIFFERENTIAL EQUATION WITH DEVIATING ARGUMENT

***Abstract**: The paper describes the construction of a shape of the orthopedic boot-tree print by means of the solution to differential equation with deviating argument. The obtained solutions to the second-order differential equation with deviating argument allow for describing the shapes of the orthopedic boot-tree print with high degree of accuracy. It also allows for varying the shapes of the orthopedic boot-tree print when moving from the one size to the second one in an unlimited number that is of particular relevance in the production of orthopedic shoes.*

***Key words**: orthopedic boot-tree; differential equation with deviating argument*

***Language**: English*

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Introduction

In the footwear industry, great attention is devoted to the issue of designing the inner shape of footwear, that is, design of boot-trees. It is well-known that from a geometrical standpoint, boot-tree has a complex shape, its description by using methods of mathematical research is quite a long and labor-intensive process. In general, the technical side of the inner shape of footwear (boot-tree) is heterogeneous. In the process of designing the internal shape of boot-trees, it is necessary to take into account data on the anthropometric sizes and shapes of foot. Proceeding from biomechanical properties of foot, it is necessary to transform the obtained parameters and, on that basis, to determine curvilinear lines of boot-tree. The development of a new algorithm describing the geometrical surface of boot-tree, and its practical application represent one of the basic and urgent problems in the footwear industry

Materials and Methods

An algorithm describing the geometrical surface of boot-tree is considered in the works of numerous researchers [1-4, 8]. To describe the geometric shape of boot-tree, they have used the following methods of research: the radius-diagram, biquadratic spline, bicubic interpolating spline (for description of transverse-ertical and horizontal sections), etc. These methods are of considerable complexity, and they require a great deal of time during the process of designing boot-trees, and they also are characterized by certain inaccuracies.

The authors of this paper have decided to describe the geometric shape of boot-tree by using the theory of differential equations with deviating argument, the issue will be resolved in a relatively brief time, and the obtained result will be far more accurate. The issue is particularly relevant when it comes to designing the orthopedic boot-trees for patients having a deformed and pathological foot.

Since the 1930s, there has been great interest of mathematicians in the theory of differential equations with deviating argument. This is attributed to the fact that the use of these equations in natural sciences and technical areas has significantly increased. A systematic study of this theory originates from the works of O. Polusukhin, E. Shmidt, F. Shearer, G. Hilb, A Myshkis, K. Voltera and other researchers [5-7, 9-10].

It is well-known that the first-order non-linear ordinary differential equation is written down as follows:

$$y'(x) = f(x, y(x)) \qquad (1)$$

If we move from this equation to differential eqaution with deviating argument, then (1) will take the following form:

$$y'(x) = f(x, y(x - \propto(x))) \qquad (2)$$

If $\alpha(x) \geq 0$, then the equation (2) is called an equation "with lagging argument", but if $\alpha(x) \leq 0$, then (2) is called an equation "with advanced argument". By means of both (1) and (2) differential equations, there are described different prcesses, but the speed of a process described for the equation (1) is determined at any moment, according to the state of this particular moment, but the speed of a process described for the equation (2) is determined in accordance with the previous state of a certain time of this process, or by the successor state of a certain time.

To illustrate the use of differential equation with deviating argument, we have to set initial condition for it, that is, to consider Cauchy problem. Our aim is to construct the shapes of the boot-tree print by means of the line integrals of the solutions to Cauchy problem for differential equations with deviating argument.

Consider differential equation "with advanced argument":

$$y'(x) = y(x+1) \qquad (3)$$

Set the initial condition for this equation:

$$y(x) = \varphi(x) \quad -\varphi < x \leq 0 \qquad (4)$$

Then from the equation (3), in sequance on the segments [0, 1], [1, 2], ..., by differentiation, it is possible to find out Y(x), by the following formula

$$y(x) = y'(x-1) \qquad (5)$$

We consider only those values of φ(x), for which, the problems (3) and (4) have the solutions. Suppose φ(x)= x. We search for a solution to the equation (3) by using the Euler method, particularly we search for the solution to (3) in the following form: $y = ce^{\lambda x}$

We shall obtain the family of the solutions in the following form:

$$y(x) = ce^{\propto x} cos\beta x \quad -\infty < x < \infty,$$

where α and β are the real numbers, and they satisfy the following equation:

$$\alpha + \beta_i = e^{\alpha + \beta_i} \quad (i = \sqrt{-1}) \qquad (6)$$

It is easy to check that the equation (6) has an infinite number of the solutions. Indeed, the equation (6) is equivalent to the following system:

$$\begin{cases} \alpha = e^{\alpha} \cos\beta \\ \beta = e^{\alpha} \sin\beta \end{cases} \qquad (7)$$

The line determined by the first equation of a system (7) on the planes α and β_{ni}, is composed of an even number of congruent "lengths", and each of them is spread out towards the positive direction of the axis α, and it has two asymptotes $\beta = k\pi \pm \frac{\pi}{2} \ (k = \pm 1, \pm 3, \pm 5, \dots)$ (See Fig. 1).

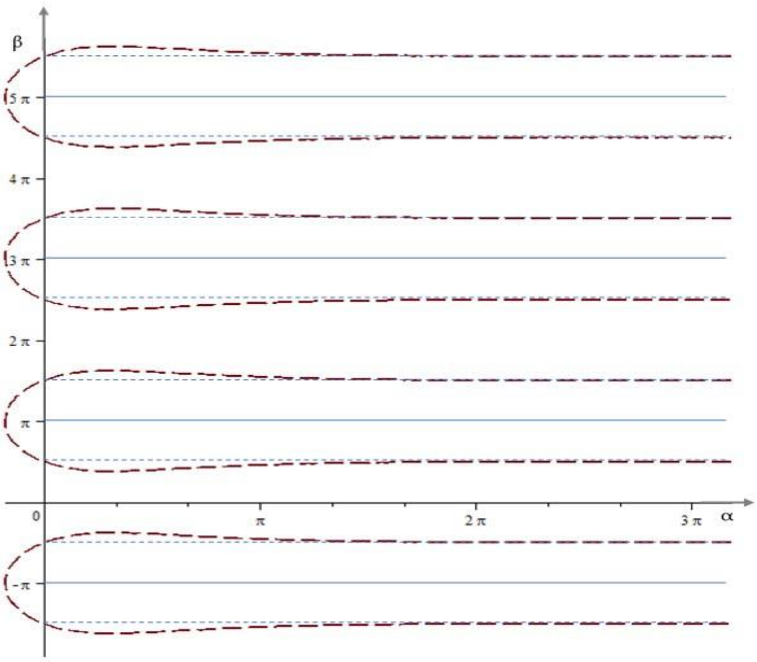

Fig. 1. Integral curves of solution of the first equation of the system (7), when
$$\boldsymbol{\beta = k\pi \pm \frac{\pi}{2} \ (k = \pm 1, \pm 3, \pm 5, \dots)}$$

The line determined by the second equation of a system (7), is composed of an even number of congruent "lengths", and each of them, except for one, is placed in the half-plane, and it has two:

$\beta = t\pi \pm \frac{\pi}{2}$ $\left(t = \pm\left(2+\frac{1}{2}\right), \pm\left(4+\frac{1}{2}\right), \pm\left(6+\frac{1}{2}\right), \dots\right)$ (See Fig. 2).

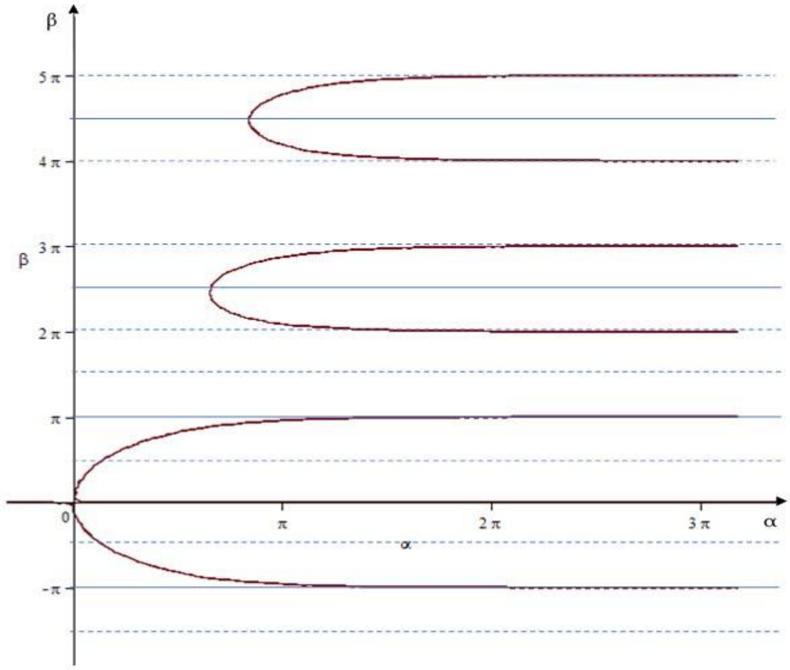

Fig. 2. Integral curves of solution of the first equation of the system (7), when
$$\boldsymbol{\beta = t\pi \pm \frac{\pi}{2} \left(t = \pm\left(2+\frac{1}{2}\right), \pm\left(4+\frac{1}{2}\right), \pm\left(6+\frac{1}{2}\right), \dots\right)}$$

As we can see, the equation (5) has an infinite number of the solutions. If we renumber these solutions, we will obtain α_{ni} and βni $(i = 1, 2, 3, \dots)$ sequences. If we equate the absolute values of both parts of a system (7), we will obtain:

$$\sqrt{a^2+b^2} = e^{\alpha}$$

Therefrom, we obtain that when $n \to \infty$, then $|\alpha_n| \to \infty$ $|\beta_n| \to \infty$, if we take the sequences of the solutions to the equations (3) and (4):

$$y_n(x) = e^{-\sqrt{\alpha_n}} e^{-\alpha_n x} \cos \beta_n x$$

Then, it clear that $\lim_{n\to\infty} \max_{-\infty \le x \le 0} |y_n(x)| = \lim_{n\to\infty} e^{-\sqrt{\alpha_n}} = 0$

Moreover, for any ε>0, we have

$$\lim_{n\to\infty} \max_{0\le x\le\varepsilon} y_n(x) = +\infty, \ \lim_{n\to\infty} \min_{0\le x\le\varepsilon} y_n(x) = -\infty$$

If we change the initial condition φ(x) = x, it is obvious that, the line integrals of the solutions to the problem (3) - (4) will change their forms. By using this method, we can obtain the desirable shape in the front part of the boot-tree print.

If we take φ(x) = ax + b as an initial condition, in this case the lines determined by the first equation of a system (7) will not be symmetrical to the lines β=(2k-1)π, but their asymptotes will be $\beta = k\pi \pm \frac{\pi}{2}$ $(k = \pm 1, \pm 2, \pm 3, \dots)$. For example, if a=2 and b=1 φ(x) = 2x + 1. In this case, the lines determined by the first equation of a system (7) will take the following form (see Fig. 3).

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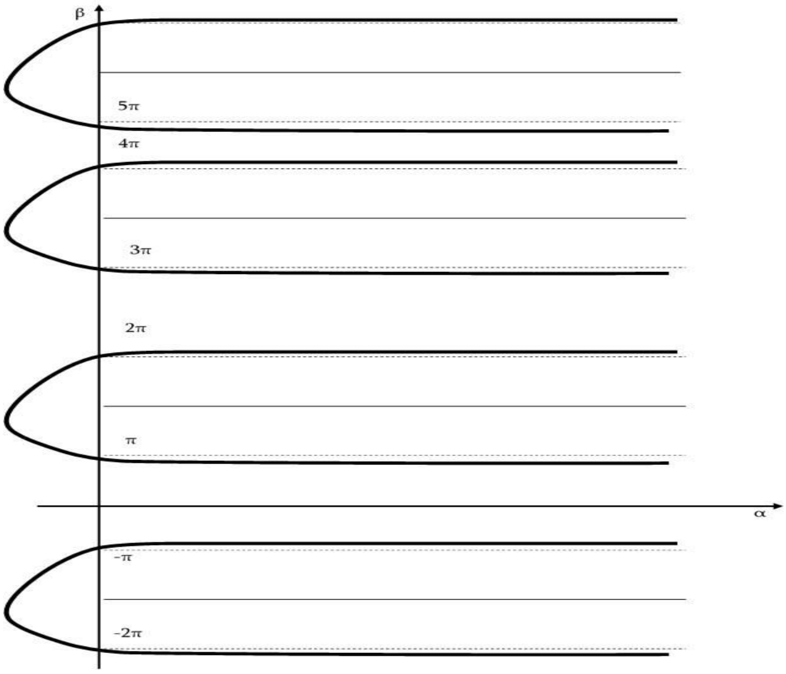

Fig. 3. Integral curves of solution of the first equation of the system (7), when a=2, b=1 and φ(x) = 2x + 1

If $a = -\frac{1}{2}$ and b=1, then $\varphi(x) = -\frac{1}{2}x + 1$. In this case, the lines determined by the first equation of a system (7) will take the following form (see Fig. 4).

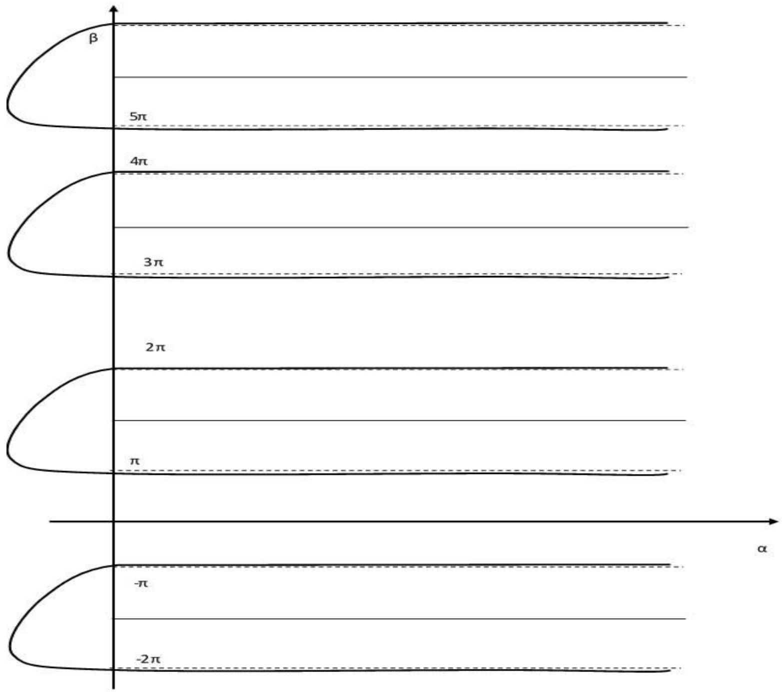

Fig. 4. Integral curves of solution of the first equation of the system (7), when $a = -\frac{1}{2}$, b=1 and $\varphi(x) = -\frac{1}{2}x + 1$

Conclusion:

The obtained solutions to the second-order differential equation with deviating argument allow for describing the shapes of the orthopedic boot-tree print with high degree of accuracy. It also allows for varying the shapes of the orthopedic boot-tree print when moving from the one size to the second one in an unlimited number. The latter is of particular relevance in the production of orthopedic shoes.

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