AXISYMMETRIC MAGNETOELASTIC SHELLS DEFORMATION
WITH ACCOUNT FOR ANISOTROPY OF CONDUCTIVE PROPERTIES

Abstract: The effect of account for external magnetic field when determining the stress-strain state of current-carrying anisotropic shells in a geometrically nonlinear statement is studied in this paper on the example of a flexible current-carrying shell located in a magnetic field. It is shown that with a change in external normal component of magnetic induction, there is a significant change in the stress state of a shell and its electromagnetic field.

Key words: shell, magnetic field, magneto elasticity.
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Introduction

In the mechanics of conjugate fields, an important place is occupied by the study of a continuous medium motion taking into account electromagnetic effects. Studies in the mechanics of coupled fields in deformable bodies have fundamental and applied nature, which makes them especially relevant. These issues were studied in [1,2,3,5,7,8,11,17,18,21,22,23, 24,25]. In modern technology, structural materials are used that are anisotropic in the undeformed state, and the anisotropy of the properties of such materials arises as a result of application of various technological processes. The nature of the shell material anisotropy is not determined entirely by its behavior as an elastic body and the anisotropy of the material can manifest itself in relation to its other physical properties, for example magnetic and dielectric permeability and electrical conductivity. Some of the most important anisotropic materials have a crystalline structure. The most characteristic feature of crystals physical properties is their anisotropy and symmetry. Due to the periodicity, regularity, and symmetry of internal structure, a number of properties are discovered in crystals that are impossible to find in isotropic bodies. The anisotropic physical properties of crystals are extremely sensitive to external influences. Therefore, selecting and combining these effects, we may create the materials with unique, unusual properties that are used in modern technology.

Problems interaction between electro-magnetic field and deformed bodies are frequent in advanced technology.

I. MATERIAL RELATIONS. BASIC EQUATIONS. By an electromagnetic field we mean a combination of four vectors: \( \mathbf{E} \) - electric field strength; \( \mathbf{H} \) - magnetic field strength; \( \mathbf{D} \) - electric induction; \( \mathbf{B} \) - magnetic induction. These vectors are assumed to be continuous ones together with their first
derivatives, functions of coordinates and time at all non-singular points. Discontinuities in field vectors or their derivatives can occur on surfaces at a sharp change in physical properties of the medium (density, conductivity, and others). Sources of electromagnetic field are electric charges, characterized by charge density $\rho_e$, and the currents, which are given by the current density vector $J$. In differential form, the laws of electromagnetism are written as

$$\text{div}D = \rho_e, \quad \text{div}B = 0, \quad \text{rot}E = -\frac{\partial B}{\partial t}, \quad \text{rot}H = J + \frac{\partial D}{\partial t}. \quad (1)$$

In material media, under external electromagnetic field, polarization and magnetization processes occur. The nature of the functional relationships between the vectors $\vec{E}$ and $\vec{B}$, as well as the vectors $\vec{H}$ and $\vec{D}$, should be determined only by physical properties of the medium itself in the immediate vicinity of this point [1,4,6,11,19,20,21]. If an elastic body is in vacuum, then for vacuum the determining relations should be fulfilled

$$\vec{D} = \varepsilon_0 \varepsilon \vec{E} ; \quad \vec{B} = \mu_0 \mu \vec{H}. \quad (2)$$

Here $\varepsilon_0, \mu_0$ are the electric and magnetic constants. In vacuum, equations (1) are satisfied under the assumption that $\vec{J} = 0$. In system SI, the numerical values of $\varepsilon_0$ and $\mu_0$ are equal to:

$$\varepsilon_0 = 8.854 \cdot 10^{-12} \approx \frac{1}{c^2} \cdot 10^{-9} \text{F/m}, \quad \mu_0 = 4\pi \cdot 10^{-7} = 1.257 \cdot 10^{-6} \text{H/m}.$$\n
Here $c_2 = 1/\varepsilon_0 \mu_0$. $c$ is the speed of light in vacuum.

For an isotropic medium $\vec{D}$ is parallel to $\vec{E}$ and not to $\vec{H} - \vec{B}$.

Usually for isotropic elastic bodies, the relationship between these vectors is linear:

$$\vec{D} = \varepsilon \vec{E} ; \quad \vec{B} = \mu \vec{H}, \quad (3)$$

where $\varepsilon = \varepsilon_0 \varepsilon_r$ and $\mu = \mu_0 \mu_r$; $\varepsilon_r$, $\mu_r$ - are the dimensionless coefficients of relative dielectric and magnetic permeability of the medium. In anisotropic media, the properties in different directions are different, and $\vec{D}$ may depend not only on $\vec{E}$. For example, in piezoelectricity, the vector of electric induction is a function of the vector of electric field strength $\vec{E}$ and the tensor of mechanical deformation.

Media with magnetic properties are called magnets. When introduced into an external magnetic field, all bodies are magnetized to one degree or another, that is, they create their own magnetic field, which is superimposed on the external field. By their magnetic properties, the magnets are divided into three main groups: ferromagnets, diamagnets and paramagnets.

The paramagnetism of metals is due to the magnetic moments of conduction electrons and crystal lattice ions. In alkali and alkaline earth metals, the magnetic moments of ions are zero, and paramagnetism is associated only with conduction electrons. In diamagnetic and paramagnetic media, the relationship between $\vec{H}$ and $\vec{B}$ is represented by formula (3), with a high degree of accuracy $\mu = \mu_0$. Strictly speaking, in stating the relation between $\vec{D} = \varepsilon \vec{E}$ and $\vec{B} = \mu \vec{H}$ it is necessary to use the analysis of atomic structure of matter, since only microscopic theory can make it possible to calculate the average field inside the body, and the local values of this field in the vicinity of individual atoms. Microscopic theory makes it possible to answer the question of how an atom will be deformed under the influence of a local field, and the total effect of atomic deformation is described using parameters $\varepsilon$ and $\mu$ (their tensor or vector analogues). In order to make the system of Maxwell equations (1) closed, it is necessary to add to two equations (3) the third one - the ratio between the density of electric current and electric field. From experimental data it is known that both for solids and for weakly ionized solutions

$$\vec{J} = \sigma \vec{E}, \quad (4)$$

where $\sigma$ is the specific electrical conductivity of the medium. Equation (4) is usually called Ohm’s Law in differential form.

The proportionality coefficient between vectors $\vec{j}$ and $\vec{E}$ in equation (4) - specific conductivity $\sigma$ - is an important characteristic of the medium. In isotropic media in the absence of an external constant magnetic field, $\sigma$ is the scalar quantity. Its value depends on temperature: with increasing temperature, specific conductivity decreases. Media can vary greatly in terms of conductivity, so their behavior in electromagnetic fields can also be completely different.

In many anisotropic media, the parameter $\sigma$ is a tensor of the second rank [4,11,20,24]:

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

In this case, the conduction current density and electric field strength in the general case do not coincide in direction. The greater the value of $\sigma$, the greater the conduction current density in the medium at the same electric field strength.

At low temperatures, many materials become ideal conductors in which $\sigma \rightarrow \infty$.

In this case, the current in the closed ring can retain its value indefinitely. To change the current, an electric field must be applied. Consider a body that moves in an external magnetic field at velocity $\vec{V}$.

In accordance with Newton’s first Law, a material point maintains a state of rest or uniform
rectilinear motion until action from other bodies takes it out of this state.

The reference frame, with respect to which a material point free from external influences is at rest or moves uniformly and rectilinearly, is called the inertial reference frame.

It is known that the Maxwell equations are invariant under the Lorentz transforms. The formulas of the Lorentz transforms for vectors \( \vec{E}, \vec{B}, \vec{D} \) and \( \vec{H} \) of electromagnetic field during the transition from a stationary inertial reference system \( K \) to another inertial reference system \( K' \) moving relative to \( K \) uniformly and rectilinearly, along, for example, the positive direction of the \( OX \) axis with velocity \( V \), have the following form in SI:

\[
E'_x = E_x, \quad E'_y = \frac{E_y - VB_z}{\sqrt{1 - V^2/c^2}}, \\
E'_z = \frac{E_z + V B_y}{\sqrt{1 - V^2/c^2}}, \\
B'_x = B_x, \quad B'_y = \frac{B_y - V E_z}{\sqrt{1 - V^2/c^2}}, \\
B'_z = \frac{B_z - V E_y}{\sqrt{1 - V^2/c^2}}, \\
D'_x = D_x, \quad D'_y = \frac{D_y - V H_z}{\sqrt{1 - V^2/c^2}}, \\
D'_z = \frac{D_z + V H_y}{\sqrt{1 - V^2/c^2}}, \\
H'_x = H_x, \quad H'_y = \frac{H_y + V D_z}{\sqrt{1 - V^2/c^2}}, \\
H'_z = \frac{H_z - V D_y}{\sqrt{1 - V^2/c^2}}.
\]

(5)

Here \( c \) is the velocity of excitation propagation in matter. The inverse transform from \( K' \) to \( K \) is obtained from given above replacement of all non-shaded quantities by shaded ones and all shaded quantities by non-shaded ones, and replacing everywhere the values of \( V \) by \(-V\). As seen from the Lorentz transforms for the electromagnetic field, similar electromagnetic fields behave differently in inertial reference frames moving relative to each other.

In particular, if in the frame of reference \( K \) there is only an electric field \( \vec{E} = E_y \hat{K} \), and \( \vec{B} = 0 \), then in the frame of reference \( K' \) both electric and magnetic fields will be observed, the vectors \( \vec{E}' \) and \( \vec{B}' \) of which are mutually perpendicular:

\[
E'_x = 0, \quad E'_y = \frac{E_y}{\sqrt{1 - V^2/c^2}} - \frac{V E_y}{c}, \quad E'_z = 0, \\
B'_x = 0, \quad B'_y = 0, \quad B'_z = -\frac{VE_y}{\sqrt{1 - V^2/c^2}}.
\]

(6)

On the contrary, if there is no electric field in the frame of reference \( K \), but only a magnetic field \( \vec{B} = B_z \hat{K} \), then in \( K' \) again magnetic and electric fields will be observed, the vectors \( \vec{B}' \) and \( \vec{E}' \) of which are mutually perpendicular:

\[
E'_x = 0, \quad E'_y = -\frac{V B_z}{\sqrt{1 - V^2/c^2}}, \quad E'_z = 0. \\
B'_x = 0, \quad B'_y = 0, \quad B'_z = -\frac{B_z}{\sqrt{1 - V^2/c^2}}.
\]

(7)

It follows from the Lorentz transforms that the scalar products of vectors \( \vec{E}' \) and \( \vec{B}' \), and \( \vec{H}' \) and \( \vec{D}' \) are invariant with respect to the selection of inertial frame of reference \( K' \) : \( \vec{E}'\hat{B}' = \vec{E}\hat{B} \) and \( \vec{H}'\vec{D}' = \vec{H}\vec{D} \).

Equations are also invariant:

\[
E'^2 - c^2 B'^2 = E^2 - c^2 B^2, \\
D'^2 - H'^2 = D^2 - H^2/c^2
\]

and material relations

\[
\vec{D}' = \varepsilon \vec{E}', \quad \vec{B}' = \mu \vec{H}', \quad \vec{J}' = \sigma \vec{E}'.
\]

In equations (7), the strokes can be omitted owing to the invariance of the Lorentz transforms, but it must be borne in mind that the material relations (8) in the moving coordinate system are transformed in accordance with (5). Next, consider the medium motion at low velocities \( V \ll c \). In this case, the relations are executed

\[
\vec{E}' = \vec{E} + \nabla \times \vec{B}, \quad \vec{B}' = \vec{B} + \frac{1}{c^2} \nabla \times \vec{H}, \\
\vec{H}' = \vec{H} - \nabla \times \vec{D}, \quad \vec{D}' = \vec{D} - \frac{1}{c^2} \nabla \times \vec{E}, \\
\vec{J}' = \vec{J} - \rho_e \vec{E}, \quad \rho_e = \rho'_e.
\]

If to substitute (9) in (8) and neglect the quantities of \( V^2/c^2 \) order, the relations are obtained in the form
\[ \vec{D} = \varepsilon \vec{E} + (\varepsilon \mu - \varepsilon_0 \mu_0) \nabla \times \vec{H}, \]
\[ \vec{B} = \mu \vec{H} - (\varepsilon \mu - \varepsilon_0 \mu_0) \nabla \times \vec{E}, \]
\[ J = \sigma \left( \vec{E} + \nabla \times \vec{B} \right) + \rho_e \vec{V}, \]

representing material relations between field vectors in a moving coordinate system through field values in a fixed system.

For many problems of magnetoelasticity, it can be assumed that in the medium \( \rho_e = 0, \varepsilon = \varepsilon_0 \). If the medium is a dia- or paramagnet, then \( \mu = \mu_0 \).

This suggests that for the considered materials the relative magnetic and dielectric permittivity coefficients \( \mu_r \) and \( \varepsilon_r \) can be considered equal to unity in a wide range of changes in magnetic induction. In this case, relations (10) have the form

\[ \vec{D} = \varepsilon \vec{E}; \quad \vec{B} = \mu \vec{H}; \quad \vec{J} = \sigma \left( \vec{E} + \nabla \times \vec{B} \right). \]

The material relationships of the electrodynamics of anisotropic conductive media can be written as

\[ D_i = \varepsilon_{ij} E_j, \quad B_i = \mu_{ij} H_j, \]
\[ J_i = \sigma_{ij} E_j, \quad (i,j) = 1,2,3 \]

Expressions (7) and (11) present a system of equations of electromagnetism for a moving homogeneous isotropic medium, and expressions (7) and (12) present a system of equations of electromagnetism for a moving anisotropic medium [1,4,7,11,20,24]. These equations hold for points in the vicinity of which physical properties of the medium change continuously.

II. PROBLEM FORMULATION.

Consider the nonlinear behavior of an orthotropic current-carrying conical shell of variable thickness \( h = 5 \cdot 10^{-4}(1 - 0.5 \frac{z}{S}) \) m. It is believed that the beryllium shell is influenced by mechanical force \( P_z = 5 \cdot 10^5 \sin \omega t \ N/m^2 \), of an external electric current \( J_{E_{CT}} = 5 \cdot 10^5 \sin \omega t \ A/m^2 \) and an external magnetic field \( B_{z0} = 0.17t \).

The electromagnetic properties of the material are characterized by tensors of electrical conductivity \( \sigma_{ij} \), magnetic permeability \( \mu_{ij} \), and dielectric constant \( \varepsilon_{ij} \).

An external electric current in an unperturbed state is uniformly distributed over the shell, i.e., the density of external current does not depend on the coordinates.

In this case, combined load consisting of the ponderomotive Lorentz force and mechanical force acts on the shell.

Suppose that the geometric and mechanical characteristics of the body are such that a version of geometrically nonlinear theory of thin shells in the quadratic approximation is applicable to describe the deformation process.

Assume that the electromagnetic hypotheses are fulfilled with respect to the electric field \( \vec{E} \) and magnetic field \( \vec{H} [1,3]. \)

These assumptions are some electrodynamic analogues of the hypothesis of undeformable normals and, together with the latter, make the hypotheses of magnetoelasticity of thin bodies.

Acceptance of these hypotheses allows reducing the problem of a three-dimensional body deformation to the problem of a coordinate surface deformation chosen arbitrarily.

The complete system of nonlinear differential equations of magnetoelasticity in the Cauchy form is taken as in [8,10,11,13,15,16,17].

III. METHODOLOGY OF THE SOLUTION

The technique for solving the magnetoelasticity problem of a truncated orthotropic spherical shell of variable thickness in an axisymmetric statement is based on the consistent use of the quasilinearization method and the discrete orthogonalization method [2,3,4, 8,10,11,13,19].

To separate the variables by the time coordinate, the implicit Newmark scheme of integration of the magnetoelasticity equations is used [19].

The next step in solving the nonlinear boundary value problem of magnetoelasticity is based on the application of the quasilinearization method, with the help of which the nonlinear boundary value problem is reduced to solving a sequence of linear boundary value problems at each time step.

Next, each of the linear boundary value problems of the sequence on the corresponding time interval is numerically solved using the stable method of discrete orthogonalization.

IV. ANALYSIS OF THE RESULTS

An orthotropic shell behavior is studied depending on the change in the external normal component of magnetic induction \( B_{z0} \). The problem for an orthotropic cone made of beryllium of variable thickness \( h = 5 \cdot 10^{-4}(1 - 0.5s/s_0) \) m is calculated under normal component of magnetic induction \( B_{z0} \), which changes as follows (8 options):

\[ B_{z0} = (-0.3, -1.0, -2.0, -3.0, -4.0, -5.0, -6.0, -7.0) \]

Boundary conditions:

\[ u = 0, \quad \omega = 0, \quad M_z = 0, \]
\[ B_{z} = B_{z0} \sin \omega \ t \quad \text{(hinged)} \quad \text{at} \quad s = s_0 = 0, \]
\[ w = 0, \quad \theta = 0, \quad N_z = 0, \]
\[ B_{z} = 0 \quad \text{(sliding)} \quad \text{at} \quad s = s_N = 0.5 M. \]

The parameters of the shell and the material are:

\[ s_0 = 0.5m, \quad h = 5 \cdot 10^{-4}(1 - 0.5s/s_0) \]
\[ m, \quad r = r_0 + s \cos \varphi, \quad r_0 = 0.5m, \rho = 2300 \text{ kg/m}^3, \]
\[ \omega = 314.16 \text{ sec}^{-1}, \quad B_{z0} = 0.17t, \quad \phi = \pi/30, \mu = \]
1.256 \cdot 10^{-6} \text{H/m}. B_{\text{go}} = 0.17, \psi = 0.03 \psi_0 = 0.09, 
J_{\text{em}} = -5 \cdot 10^{6} \sin \omega t / m^2, 
\sigma_1 = 0.279 \cdot 10^{8} (\Omega \times m)^{-1}, \sigma_2 = 0.321 \cdot 10^{8} (\Omega \times m)^{-1}, 
\sigma_3 = 1.136 \cdot 10^{8} (\Omega \times m)^{-1}. 
\varepsilon_{\psi} = 28.8 \cdot 10^{10} N/m^2, \varepsilon_\omega = 33.53 \cdot 10^{10} N/m^2, p_\xi = 5 \cdot 10^{3} \sin \omega t / N/m^2. 
The solution is found in the time interval \tau = 0 \div 10^{-2} \text{sec} for the integration time step is chosen to be \Delta t = 1 \cdot 10^{-3} \text{sec}. Maximum values obtained at time step t = 5 \cdot 10^{-2} \text{sec}. Note that in the case under consideration, the anisotropy of electrical resistivity of beryllium is \nu_3/\eta = 4.07.

Figure 1 shows the change in internal magnetic induction of a shell depending on the change in external magnetic induction at \( t = 5 \cdot 10^{-3} \text{s} \) and \( s = 0.45 \text{ m} \) for all variations of \( B_{\text{go}} \). As follows from the figure, the internal magnetic induction of a shell substantially depends on the external magnetic induction. In the considered range of changes in external magnetic induction, the internal magnetic induction reaches its maximum value at \( B_{\text{go}} = -4.0 \).

![Figure 1](image)

It was revealed that with an increase in external magnetic induction, the stresses on the outer surface of a shell vary depending on the change in the ponderomotive Lorentz force direction and the interaction with mechanical load. In the considered case, the stress on the outer surface of a shell reaches its maximum value at \( B_{\text{go}} = -4.0 \). As the value of external magnetic induction increases, the stress along the inner surface of a shell decreases too.

**V. CONCLUSION**
The coupled problem of magnetoelasticity for a flexible anisotropic shell is discussed in the paper taking into account the anisotropy of conductive properties. The results of numerical example are presented.

The analysis of the stress state of a flexible shell under time-varying mechanical force and time-varying external electric current is fulfilled taking into account mechanical and electromagnetic orthotropy. The effect of external magnetic induction on the stress state of an orthotropic shell in a geometrically nonlinear statement is analyzed. It was found that with an increase in external magnetic field induction, the induction of internal magnetic field increases too.

**References:**


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