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# ON THE ACTION OF MOBILE LOADS ON AN UNINTERRUPTED CYLINDRICAL TUNNEL

***Abstract**: He article gives the basics of the calculation technique for the action of mobile loads of long underground transport structures such as tunnels and pipelines taking into account the influence of the earth's surface. On elastic models, the dynamic behavior of unreinforced and reinforced structures at different depths of bedding is considered, as well as the effect of the type and parameters of the running load on the stress-strain state of the rock massif. The speed of the movement of the cargo is considered subsonic, which corresponds to the modern speeds of transport in the investigated underground objects. To describe the motion of a half-space and thick-walled shells, dynamic equations of the theory of elasticity in displacement potentials are used, and for thin-walled shells the classical equations of the theory of thin shells are used. Equations are written in a moving coordinate system associated with the load. A closed system of differential equations is constructed. The system of differential equations is solved by the method of separated variables, integral Fourier transforms, the Romberg, Muller and Gauss method. From the analysis of the obtained numerical results it follows that in these cases the reinforcement of the tunnel leads to a decrease in the dynamic effect of the moving loads on the earth's surface. The Earth's surface has an uneven effect on the stress-strain state of the rock massif under the action of moving loads. For a load with a shorter period, this effect is almost not noticeable, and becomes noticeable for very small periods.*

***Key words**: thick-walled shell, stationary load, cavity, mobile coordinate system, Lame potentials, Muller and Gauss method.*
***Language**: English*
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**Introduction**

In many cities it is planned to build underground highways of considerable length, as well as tunnels for new high-speed transport. Extremely widespread development of the construction of underground main pipelines providing transportation of virtually the entire volume of produced natural gas, most of the oil and various cargoes. Modern transport underground structures in accordance with the requirements of reliability and durability are among the most important objects of underground construction. Along with the static calculation of such structures [3] their dynamic calculation [1, 2] is necessary. Among the dynamic loads and impacts on underground structures in the form of tunnels and pipelines, operational transport loads and the impact of seismic waves of natural or artificial origin should be singled out. Difficulties in the calculation of objects in the presence of mobility of the load multiply increase in comparison with the volume of static calculations. Especially great mathematical difficulties appear when taking into account the massiveness of the driving loads. The study of the dynamics of extended underground structures under the action of various perturbations leads to the solution of boundary value problems in the mechanics of continuous media. [4-6]

Work in this direction with a sufficiently detailed bibliography can be found in monographs [9, 11, 12, 15] and many other publications are devoted to a generalization and systematization of research results on a comprehensive study of the dynamic behavior of cylindrical shells of various designs. The stationary solution of the dynamics of an infinitely long thin cylindrical shell immersed in an acoustic medium and subjected to an axisymmetric load moving with a constant velocity in the axial direction was investigated [13], the reaction of an infinitely long cylindrical shell in an acoustic medium to the action of a moving stepped plane shock wave was considered. The solution is given in generalized coordinates without taking into account the extension of the middle surface of the shell. In [14], such problems are solved by the method of integral transformations. Later, hinged-supported shells were considered in [10], the nonlinear dynamics of shells was investigated. In [8], the ax symmetric vibrations of a priestesses shell were studied under the action of a moving force, where the Bubnov - Galerkin method was applied to geometric coordinates and the Bogolyubov - Mitropolsky coordinate in time coordinate. Starting from the equation of shell motion [5], we studied the dynamics of a priestess's cylinder under the action of two types of loads: a concentrated normal force moving along a circle at a constant velocity, and a point wise normal force moving along the axis of the cylinder.

An approximate model approach for determining vibrations on a free surface from moving loads in reinforced tunnels of a rectangular and circular profile has been applied [7]. The action of a mobile periodic load on a circular cylindrical cavity in an elastic half-space for subsonic speeds of load motion was considered in [14] where the motion of a half-space described the dynamic equations of the theory of elasticity [5] in Lame potentials. To solve problems in this paper, a model research method is used.

The tunnel is modeled as an infinitely long circular cylindrical cavity located in a homogeneous and isotropic linearly elastic half-space parallel to its horizontal boundary. The cavity can be supported by a homogeneous or layered elastic shell (in which case the tunnel can be considered as an underground pipeline). The no stationary load acts on the surface of the cavity or on the inner surface of the shell reinforcing cavity. The speed of the load is assumed to be subsonic.

**2. Statement of the problem for a circular tunnel.**

Using the model approach for research, we will represent the tunnel as an infinitely long circular cylindrical cavity with a radius $r = R$, located in a linear viscoelastic, homogeneous and isotropic half-space $x \leq h$ (Figure 1) parallel to its horizontal boundary (the earth's surface). We define the reaction of a half-space on a moving with a constant subsonic velocity c along the cavity surface in the direction of the Z-axis of the load P.

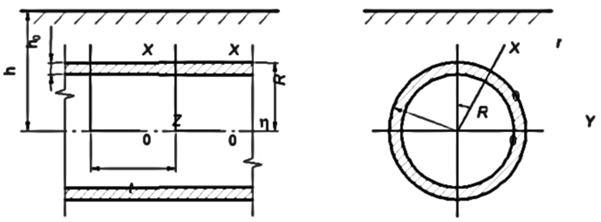


**Figure 1. The calculated scheme of the reinforced tunnel and underground pipeline**

For this, we use the equations of motion of an elastic medium in vector form [16, 17]

$$\tilde{\mu}_j \nabla^2 \bar{u} + (\tilde{\lambda}_j + \tilde{\mu}_j) grad.di\vartheta\, \bar{u} = \rho_j \frac{\partial^2 \bar{u}}{\partial t^2}, \qquad (1)$$

Here $\vec{u}(u_x, u_y, u_z)$ - vector of displacement of points of the medium; $\rho_j$ - material density; $u_x, u_y, u_z$ - displacement components; $\nu_j$ - is the Poisson's ratio;

$$\tilde{\lambda}_j = \frac{\nu_j \tilde{E}_j}{(1+\nu_j)(1-2\nu_j)}; \quad \tilde{\mu}_j = \frac{\nu_j \tilde{E}_j}{2(1+\nu_j)},$$

where $\tilde{E}_j$ – Operational modulus of elasticity, which have the form [25, 26].

$$\tilde{E}_j \phi(t) = E_{0j}\left[\phi(t) - \int_0^t R_{Ej}(t-\tau)\phi(t)\,d\tau\right] \quad (2)$$

$\phi(t)$- arbitrary time function; $R_{Ej}(t-\tau)$- relaxation core; $E_{0j}$- instantaneous modulus of elasticity; We assume the integral terms in (5) to be small, then the functions $\phi(t) = \psi(t)e^{-i\omega_R t}$, where $\psi(t)$- a slowly varying function of time, $\omega_R$ - real constant. Further, applying the freezing procedure [14], we note relations (2) as approximations of the form

$$\bar{E}\phi = E\left[1 - \Gamma^C(\omega_R) - i\Gamma^S(\omega_R)\right]\phi,$$

where

$$\Gamma^C(\omega_R) = \int_0^\infty R(\tau)\cos\omega_R\tau\,d\tau,$$

$$\Gamma^S(\omega_R) = \int_0^\infty R(\tau)\sin\omega_R\tau\,d\tau$$

respectively, the cosine and sine Fourier images of the relaxation core of the material. As an example of a viscoelastic material, we take three parametric relaxation nuclei $R(t) = Ae^{-\beta t}/t^{1-\alpha}$.

On the influence function $R(t-\tau)$ the usual requirements of inerrability, continuity (except for $t = \tau$), sign of uncertainty and monotony:

$$R > 0, \quad \frac{dR(t)}{dt} \le 0, \quad 0 < \int_0^\infty R(t)dt < 1.$$

$\vec{u}$ - the vector of displacements of the environment.

Since the steady-state process is considered, the strain pattern is stationary with respect to the moving load. Therefore, it is convenient to move to a moving coordinate system η = *z* - *ct*, connected with the load P.

Then equation (1) can be rewritten in the form

$$\left(\frac{1}{M_p^2} - \frac{1}{M_s^2}\right)\text{grad div}\,\mathbf{u} + \frac{1}{M_s^2}\nabla^2\mathbf{u} = \frac{\partial^2\mathbf{u}}{\partial\eta^2}. \quad (3)$$

Here $M_p = c/c_p$, $M_s = c/c_s$ - Mach numbers; $c_p = \sqrt{(\bar{\lambda} + 2\bar{\mu})/\rho}$, $c_s = \sqrt{\bar{\mu}/\rho}$ – complex propagation velocities of expansion waves - compression and shear in a medium.

## 3. Tasks of the action of mobile loads on an Unreinforced tunnel.

In the theoretical aspect, the solution was based on the papers [23, 24] In [25], the first and second boundary-value problems of the theory of elasticity for a half-plane with a point source of stationary waves concentrated within it, the potential of which is represented in terms of cylindrical functions, are solved by the method of expanding potentials on plane waves. And in [24] using this approach, the problem of the stationary load on the contour of a circular hole in a half-space was solved. Using the idea of these papers on the superposition of solutions and the re-expansion of plane waves into series in cylindrical functions, in [25], in contrast to the exact analytical solution for the subsonic case, when the velocity of a moving load is less than the velocity of the Rayleigh waves.

Since the steady-state process is considered, the strain pattern is stationary with respect to the moving load. Therefore, it is convenient to move to the mobile coordinate system $\eta = z - ct$, connected with the load *P*.

Then equation (1) can be rewritten in the form

$$\left(\frac{1}{M_p^2} - \frac{1}{M_s^2}\right)\text{grad div}\,\mathbf{u} + \frac{1}{M_s^2}\nabla^2\mathbf{u} = \frac{\partial^2\mathbf{u}}{\partial\eta^2}. \quad (4)$$

When the load acts on the cavity surface, we have

$$\sigma_{rj}\big|_{r=R} = P_j(\theta,\eta), \quad j = r, \theta, \eta, \quad (5)$$

where $\sigma_{rj}$ – components of the stress tensor in a medium, $P_j(\theta,\eta)$ – components of the intensity of the mobile load $P(\theta,\eta)$.

Since the boundary of the half-space is free from loads, $x = h$

$$\sigma_{xx} = \sigma_{xy} = \sigma_{x\eta} = 0. \quad (6)$$

We transform equation (1) by expressing the displacement vector of an elastic medium through Lame potentials

$$\mathbf{u} = \text{grad}\,\varphi_1 + \text{rot}\,\boldsymbol{\psi} \quad (7)$$

Potential $\boldsymbol{\psi}$ can be represented in the form [27]

$$\boldsymbol{\psi} = \varphi_2\mathbf{e}_\eta + \text{rot}(\varphi_3\mathbf{e}_\eta), \quad (8)$$

where $\mathbf{e}_\eta$ «ort axis η».

With this in mind, (5) takes the form

$$\mathbf{u} = \text{grad div}\,\varphi_1 + \text{rot}(\varphi_2\mathbf{e}_\eta) + \text{rot rot}(\varphi_3\mathbf{e}_\eta). \quad (9)$$

It follows from (3) and (8) that the potentials $\varphi_j$ satisfy the modified wave equations

$$\nabla^2\varphi_j = M_j^2\frac{\partial^2\varphi_j}{\partial\eta^2}, \quad j = 1, 2, 3. \quad (10)$$

Here $M_1 = M_p$, $M_2 = M_3 = M_s$.

We express the components of the stress and displacement of the material point through the potentials $\varphi_j$.

The components of the vector u (7) in cylindrical (8) and Cartesian (9) coordinate systems [24-26]:

$$u_r = \frac{\partial\varphi_1}{\partial r} + \frac{1}{r}\frac{\partial\varphi_2}{\partial\theta} + \frac{\partial^2\varphi_3}{\partial\eta\partial r},$$

$$u_\theta = \frac{1}{r}\frac{\partial\varphi_1}{\partial\theta} - \frac{\partial\varphi_2}{\partial r} + \frac{1}{r}\frac{\partial^2\varphi_3}{\partial\eta\partial\theta}, \quad (11)$$

$$u_\eta = \frac{\partial \varphi_1}{\partial \eta} + m_s^2 \frac{\partial^2 \varphi_3}{\partial \eta^2};$$
$$u_x = \frac{\partial \varphi_1}{\partial x} + \frac{\partial \varphi_2}{\partial y} + \frac{\partial^2 \varphi_3}{\partial x \partial \eta},$$
$$u_y = \frac{\partial \varphi_1}{\partial y} - \frac{\partial \varphi_2}{\partial x} + \frac{\partial^2 \varphi_3}{\partial y \partial \eta},$$
$$u_\eta = \frac{\partial \varphi_1}{\partial \eta} + m_s^2 \frac{\partial^2 \varphi_3}{\partial \eta^2},$$

Where $m_s^2 = 1 - M_s^2$.

Volumetric strain

$$\varepsilon = \mathrm{div}\,\mathbf{u} = \nabla^2 \varphi_1. \tag{12}$$

Using Hooke's law, taking into account (9), (11), we find expressions for the stress tensor components in cylindrical and Cartesian coordinates

$$\sigma_{\eta\eta} = (2\mu + \lambda M_p^2)\frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu m_s^2 \frac{\partial^3 \varphi_3}{\partial \eta^3},$$
$$\sigma_{\theta\theta} = \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + \frac{2\mu}{r}\left(\frac{1}{r}\frac{\partial^2 \varphi_1}{\partial \theta^2} + \frac{\partial \varphi_1}{\partial r} + \frac{1}{r}\frac{\partial \varphi_2}{\partial \theta} - \frac{\partial^2 \varphi_2}{\partial r \partial \theta} + \frac{1}{r}\frac{\partial^3 \varphi_3}{\partial \theta^2 \partial \eta} + \frac{\partial^2 \varphi_3}{\partial r \partial \eta}\right),$$
$$\sigma_{rr} = \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu\left(\frac{\partial^2 \varphi_1}{\partial r^2} + \frac{1}{r}\frac{\partial^2 \varphi_2}{\partial \theta \partial r} - \frac{1}{r^2}\frac{\partial \varphi_2}{\partial \theta} + \frac{\partial^3 \varphi_3}{\partial r^2 \partial \eta}\right),$$
$$\sigma_{r\eta} = \mu\left(2\frac{\partial^2 \varphi_1}{\partial \eta \partial r} + \frac{1}{r}\frac{\partial^2 \varphi_2}{\partial \theta \partial \eta} + (1 + m_s^2)\frac{\partial^3 \varphi_3}{\partial \eta^2 \partial r}\right),$$
$$\sigma_{\eta\theta} = \mu\left(\frac{2}{r}\frac{\partial^2 \varphi_1}{\partial \theta \partial \eta} - \frac{\partial^2 \varphi_2}{\partial r \partial \eta} + \frac{(1 + m_s^2)}{r}\frac{\partial^3 \varphi_3}{\partial \theta \partial \eta^2}\right),$$
$$\sigma_{r\theta} = 2\mu\left(\frac{1}{r}\frac{\partial^2 \varphi_1}{\partial \theta \partial r} - \frac{1}{r^2}\frac{\partial \varphi_1}{\partial \theta} - \frac{\partial^2 \varphi_2}{\partial r^2} - \frac{m_s^2}{2}\frac{\partial^2 \varphi_2}{\partial \eta^2} + \frac{1}{r}\frac{\partial^3 \varphi_3}{\partial r \partial \eta \partial \theta} - \frac{1}{r^2}\frac{\partial^2 \varphi_3}{\partial \eta \partial \theta}\right);$$
$$\sigma_{\eta\eta} = (2\mu + \lambda M_p^2)\frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu m_s^2 \frac{\partial^3 \varphi_3}{\partial \eta^3},$$
$$\sigma_{yy} = \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu\left(\frac{\partial^2 \varphi_1}{\partial y^2} - \frac{\partial^2 \varphi_2}{\partial x \partial y} + \frac{\partial^3 \varphi_3}{\partial y^2 \partial \eta}\right),$$
$$\sigma_{xx} = \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu\left(\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial x \partial y} + \frac{\partial^3 \varphi_3}{\partial x^2 \partial \eta}\right),$$
$$\sigma_{x\eta} = \mu\left(2\frac{\partial^2 \varphi_1}{\partial \eta \partial x} + \frac{\partial^2 \varphi_2}{\partial y \partial \eta} + (1 + m_s^2)\frac{\partial^3 \varphi_3}{\partial \eta^2 \partial x}\right),$$
$$\sigma_{\eta y} = \mu\left(2\frac{\partial^2 \varphi_1}{\partial y \partial \eta} - \frac{\partial^2 \varphi_2}{\partial x \partial \eta} + (1 + m_s^2)\frac{\partial^3 \varphi_3}{\partial y \partial \eta^2}\right),$$
$$\sigma_{xy} = 2\mu\left(\frac{\partial^2 \varphi_1}{\partial x \partial y} - \frac{\partial^2 \varphi_2}{\partial x^2} - \frac{m_s^2}{2}\frac{\partial^2 \varphi_2}{\partial \eta^2} + \frac{\partial^3 \varphi_3}{\partial x \partial y \partial \eta}\right).$$

Thus, to determine the components of the stress-strain state of the medium, it is necessary to solve equations (9) together with the boundary conditions.

In cases where circular tunneling or underground pipelines are thin-walled structures, the considered model of the tunnel can be adopted as a design model, with the reinforcement of the cavity by a thin elastic cylindrical shell of thickness $h_0$ (Figure 1). Because of the small thickness of the shell, we assume that the surrounding array is in contact with the shell along its median surface. The load P, moving with a constant subsonic speed c in the direction of the Z-axis, acts on the inner surface of the shell.

To describe the motion of the shell, we use the classical equations of the theory of thin shells [21]

$$\frac{\partial^2 u_{0z}}{\partial z^2} + \frac{1-\nu_0}{2R^2}\frac{\partial^2 u_{0z}}{\partial \theta^2} + \frac{1+\nu_0}{2R}\frac{\partial^2 u_{0\theta}}{\partial z \partial \theta} + \frac{\nu_0}{R}\frac{\partial u_{0r}}{\partial z} = \rho_0 \frac{1-\nu_0}{2\mu_0}\frac{\partial^2 u_{0z}}{\partial t^2} + \frac{1-\nu_0}{2\mu_0 h_0}(P_z - q_z),$$
$$\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0z}}{\partial z \partial \theta} + \frac{(1-\nu_0)}{2}\frac{\partial^2 u_{0\theta}}{\partial z^2} + \frac{1}{R^2}\frac{\partial^2 u_{0\theta}}{\partial \theta^2} + \frac{1}{R^2}\frac{\partial u_{0r}}{\partial \theta} = \rho_0 \frac{1-\nu_0}{2\mu_0}\frac{\partial^2 u_{0\theta}}{\partial t^2} + \frac{1-\nu_0}{2\mu_0 h_0}(P_\theta - q_\theta),$$
$$\frac{\nu_0}{R}\frac{\partial u_{0z}}{\partial z} + \frac{1}{R^2}\frac{\partial u_{0\theta}}{\partial \theta} + \frac{h_0^2}{12}\nabla^2\nabla^2 u_{0r} + \frac{u_{0r}}{R^2} = -\rho_0 \frac{1-\nu_0}{2\mu_0}\frac{\partial^2 u_{0r}}{\partial t^2} - \frac{1-\nu_0}{2\mu_0 h_0}(P_r - q_r) \tag{13}$$

where $u_{0z}$, $u_{0\theta}$, $u_{0r}$ are the displacements of the points of the middle surface of the shell; $P_z$, $P_\theta$, $P_r$ - components of the intensity of the mobile load P;

$$q_z = \sigma_{rz}\big|_{r=R}, \quad q_\theta = \sigma_{r\theta}\big|_{r=R}, \quad q_r = \sigma_{rr}\big|_{r=R} \text{-}$$

components of the reaction surrounding the shell environment; $\nu_0$, $\mu_0$, $\rho_0$ are the Poisson's ratio, the shear modulus and the density of the shell material, respectively. In the moving coordinate system, equations (13) are rewritten in the form

$$\left[1-\frac{(1-\nu_0)\rho_0c^2}{2\mu_0}\right]\frac{\partial^2u_{0\eta}}{\partial\eta^2}+\frac{1-\nu_0}{2R^2}\frac{\partial^2u_{0\eta}}{\partial\theta^2}+\frac{1+\nu_0}{2R}\frac{\partial^2u_{0\theta}}{\partial\eta\partial\theta}+\frac{\nu_0}{R}\frac{\partial w_{0r}}{\partial\eta}=\frac{1-\nu_0}{2\mu_0h_0}\left(P_\eta-q_\eta\right),$$

$$\frac{1+\nu_0}{2R}\frac{\partial^2u_{0\eta}}{\partial\eta\partial\theta}+\frac{(1-\nu_0)}{2}\left(1-\frac{\rho_0c^2}{\mu_0}\right)\frac{\partial^2u_{0\theta}}{\partial\eta^2}+\frac{1}{R^2}\frac{\partial^2u_{0\theta}}{\partial\theta^2}+\frac{1}{R^2}\frac{\partial w_{0r}}{\partial\theta}=\frac{1-\nu_0}{2\mu_0h_0}\left(P_\theta-q_\theta\right),$$

$$\frac{\nu_0}{R}\frac{\partial u_{0\eta}}{\partial\eta}+\frac{1}{R^2}\frac{\partial u_{0\theta}}{\partial\theta}+\frac{h_0^2}{12}\nabla^2\nabla^2w_{0r}+\frac{(1-\nu_0)\rho_0c^2}{2\mu_0}\frac{\partial^2w_{0r}}{\partial\eta^2}+\frac{w_{0r}}{R^2}=-\frac{1-\nu_0}{2\mu_0h_0}\left(P_r-q_r\right)$$

The motion of the half-space is described by the dynamic equations of elasticity theory in Lame potentials.

Let's consider two cases of conjugation of a shell with an environment: rigid and sliding. In these cases, the boundary conditions have the form:

- At sliding contact

$$\sigma_{rj}\big|_{r=R}=0,\ j=\eta,\theta,\quad w_r\big|_{r=R}=w_{0r}, \tag{14,a}$$

- At hard contact

$$u_j\big|_{r=R}=u_{0j},\ j=\eta,\theta,r\ . \tag{14,б}$$

Thus, in this formulation, in order to determine the components of displacements and stresses of the medium, it is necessary to jointly solve Esq. (13), subject to the boundary conditions (14), depending on the conjugation condition of the shell with the medium.

In the moving coordinate system, we apply to the equations of motion and the boundary conditions a complex Fourier transform of the form [24-26].

$$\overline{\varphi}(\xi)=\int_{-\infty}^{\infty}\varphi(\eta)e^{-i\xi\eta}d\eta, \tag{15}$$

$$\varphi(\eta)=-\frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{\varphi}(\xi)e^{i\xi\eta}d\xi\ .$$

Writing general solutions of the transformed equations of motion of the tunnel in the form (4) - (15), we find the following system of algebraic equations for determining the dimensionless transform ants of displacements of an intermediate surface

$$-\xi^2U_0+i\xi G_1w_0=-\zeta^2\frac{1-G_1}{3}G_0^2U_0;$$

$$i\xi G_1U_0-\frac{1-G_1}{3}G_0^2\zeta^2w_0+\left(1+\frac{k^2\zeta^4}{12}\right)w_0-$$

$$-\frac{1-G_1}{2k}\frac{\zeta G_{11}}{\gamma}w_0=C_2P_{10}.$$

Where

$$G_1=G_2/G_1;\ k=h/a;\ P_{10}=P_0a/Eh;$$

$$\{U_0,\ \ W_0\}=\frac{1}{h}\{U_1,\ \ W_1\};\quad C_0^2=C\left(\frac{3}{2}\frac{\rho_1}{G_1}\right);$$

The stress at the boundary of the soft layer and elastic among (r = b) in the dimensionless form has the form:

$$\sigma_{rr}^*=\int\frac{4}{\pi}\left\{-\frac{(1-\eta)H_1^{(1)}(\overline{\alpha}a)}{\delta_1}\sin\theta+\sum^{\infty}\frac{i^{n+1}H_n(\overline{\alpha}\ a)}{\Delta n}\sin n\theta\right\}e^{i\xi\eta}d\xi$$

$$\sigma_{r\theta}^*=\int\frac{2}{\pi}\left\{-\frac{i\ \overline{\beta}a^2}{\overline{\beta}^3a^3H_1^{(1)}(\overline{\beta}a)+8\eta\left(\frac{\overline{\beta}^2a^2}{2}H^{(1)}{}_0(\beta a)-\overline{\beta}aH_1(\beta a)\right)}-\right.$$

$$-\frac{2}{\delta_1}\left[(1+\eta)H_1(\overline{\alpha}\ a)-\overline{\alpha}\ aH_0(\overline{\alpha}\ a)cos\theta\right]-$$

$$\left.-2\sum_{n=2}^{\infty}\frac{i^{n+1}\left[-nH_n^{(1)}(\overline{\alpha}\ a)+(\overline{\alpha}\ a)H_{n-11}(\overline{\alpha}\ a)\right]}{\Delta n}\cos n\theta\ \right\}e^{i\xi\eta}$$

where

$$\delta_1=-4\eta H_1^{(1)}(\overline{\alpha}\ a)H_1(\overline{\beta}\ a)+(1+\eta)\overline{\alpha}\ aH_0(\overline{\alpha}\ a)+H_0(\overline{\beta}\ a),$$

$$\Delta n=n\overline{\alpha}\ aH_{n-1}(\overline{\alpha}\ a)H_{n1}(\overline{\beta}\ a)+n(\overline{\beta}\ a)H_{n-1}(\overline{\beta}\ a)H_n(\overline{\alpha}\ a)-$$

$$-\overline{\alpha}\overline{\beta}a^2H_{n-1}^{(1)}(\overline{\alpha}\ a)H_{n-1}^{(1)}(\overline{\beta}\ a)$$

Here δ = c/c$_B$ is the ratio of the density of the environment to the density of the soft layer; $\overline{\alpha},\overline{\beta}$ - are functions of ξ and η.

We find the following expression for the load transformer, which is transferred to the shell from the side of the soft layer

$$\overline{q}_{rc}=-G_1\frac{\xi}{q}C_1w_0-C_2P_0(\xi);$$

$$C_1=\sum_{j=1}^{4}\frac{A_{4j}\big|_{k_{c1}=0}B_{3j}}{\det\|A_{ke}\|};$$

$$C_2=\sum_{j=1}^{4}\frac{(-1)^jA_{4i}\big|_{k_{c1}=0}B_{3j}}{\det\|A_{ke}\|}.$$

Elements of the determinant $\det\|A_{ke}\|$ is computed then formula $A_{11}=-2M_1;\quad A_{12}=-a_{11};$

$$A_{13}=nM_{12};\ A_{14}=-A_{13};\ A_{21}=-S_1A_{11};$$

$$A_{22}=A_{12}*k_0(z_1)/k_1(z_1);$$

$$A_{23}=A_{13}*I_0(z_2)/I_2(z_2);$$

$$A_{24}=A_{13}*k_0(z_1)/k_1(z_2);\ A_{31}=\frac{1}{2}A_{11};\ A_{32}=-\frac{1}{2}A_{11};$$

$$A_{41}=n_1k_0(z_3)/k_1(z_4)-2A_{21}/(z_3/M_2);\ A_{31}=A_{13}/n_1;$$

$$A_{34}=-A_{13}/n_1;$$

$$A_{42}=n_1I_0(z_3)/I_1(z_4)-2M_1S_1(z_3)/I_1(z_4);$$

$$A_{43}=-2M_{12}^2\left(k_0(z_5)/k_1(z_6)+I_1(z_5)/I_1(z_6)/(z_6/M_2)\right);$$
$$A_{44}=-2M_{12}^2\left(I_0(z_5)/k_1(z_6)+I_1(z_5)/k_1(z_6)/(z_6/M_2)\right);$$

Where is $m_1=\sqrt{1-M_p^2};\ m_{12}=\sqrt{1-M_S^2};$

$$z_1=M_1\eta;\quad z_2=M_{121}\eta;\quad z_3=M_1\eta;$$
$$z_4=m_1\eta(1+k_{11});\quad z_5=m_1\eta;\quad k_{11}=(b-a)/a;$$

$k_{1_0}$ $k_1$ - Modified Neumann functions; $I_{1_0}$ $I_1$ - modified Bessel functions; the general solution of the equations of the motion of the environment has the form

$$(C_f<C_S<C_p)$$

$$\varphi(r,\xi)=A_n(\xi)k_n(m_1\xi r)+B_n(\xi)I_4(m_1\xi r)$$
$$\psi(r,\xi)=C_n(\xi)k_n(m_{i2}\xi r)+D_n(\xi)I_4(m_{12}\xi r)\,. \quad (16)$$

His expression for the original transform of the normal displacement has the form

$$w_0=-\frac{1-\nu}{m}\sum_{i=o}^{\infty}\left\{\int_{-\infty}^{\infty}\frac{\Delta_1[a\cos(\zeta\eta)-\zeta\lim(\zeta\eta)]d\xi}{[a^2+\zeta^2]\det|A_{ke}|}\right\} \quad (17)$$

Defining $\Delta_j$ $(j=1.2.......5)$ is obtained from $\det\|A_{ke}\|$ by replacing j = 20 by the column C with the elements {0; 0; 1; 0; 0}.

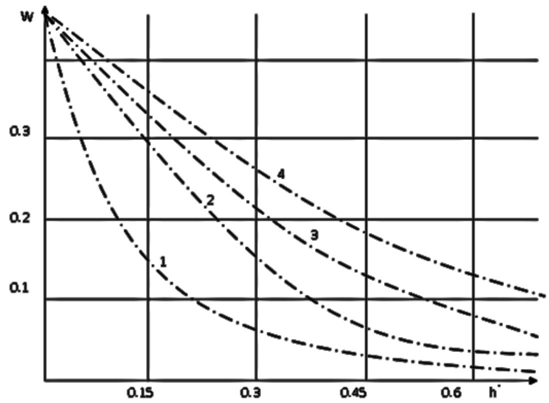


**Fig. 2. Shell deflections as a function of thickness.**

After this function $A(\zeta)......D(\zeta)$ from (16) can be calculated from formulas

$$\{A,B,C,D\}=\frac{a^2}{\xi^2\det\|A_{ke}\|}\left\{\frac{A_1^1}{k_1(m_1\xi)};\quad -\frac{A_2^1}{I_1(m_2\xi)};\quad -i\frac{aA_3^1}{\zeta k_1(m_{12}\xi)};\quad i\frac{aA_4^1}{\zeta I_1(m_{12}\xi)}\right\}$$
$$A_j^1=\frac{\xi}{a}M_{3k}w_0+P_0|m_{4k}|G_1(\kappa = 1,.......4)$$

$m_{ie}$ – minors of the element Aje. For a specific value of the load velocity C, the denominators under the integral expressions in formulas (14) are transcendental functions with respect to $\zeta$ C real coefficients depending on C, as well as on the mechanical parameters of the shell and the layer. Analysis of the integrals of treatment must begin with consideration of cases [25] $D(\xi,C_0)=0,$ which is equivalent to the construction of the dispersion relation in the corresponding problem of propagation of free waves and the determination of the denominator from the dispersion curves of the roots for the chosen velocity of the load C. at C< $C_5$ are possible for cases. Figure 2 shows the change in the movement of the filler, depending on the thickness of the bodies for different values of the rigidity of the aggregate. As can be seen from the drawing (γ = 100, 50,10,2), that for a sufficiently rigid layer (γ = 100), the deflections of the shell essentially decrease [18-20, 21, 22]. For a given speed C, there are one or two different denominator roots. For some values of *C*, the denominator has a double root. This case corresponds to a minimum of the corresponding dispersion curve in Fig. Such a velocity is called resonance and is denoted by $C^{\alpha}$.

A resonance effect appears, or which deflections and contact pressures tend to infinity. For a given value of C, the denominator has no roots on the real axis, as seen in Figure 2, this will be either, C<$C_ф$ (up to resonance mode). At this speed of motion, the inversion integrals are not special and can be found by effective numerical methods.

Dividing the integral (17) into two terms

$$w_0=\frac{1}{\pi}\int_0^{\infty}x_1(\Omega)d\Omega$$

and

$$w_0 = \frac{1}{\pi}\int_{\omega_1}^{\omega_2} x_1(\Omega)d\Omega \qquad (18)$$

The value of the integral (18) was found by the numerical method [23]. When the integral is calculated by the Romberg method, it is necessary to repeatedly calculate the integrand function. The inverse Fourier transform (29) was numerically fulfilled. It is shown that at an integration step of 1.01, the error of the procedure does not exceed 0.3-0.5%.

**4. Conclusions.**

1. From the analysis of these results it follows that for any conjugation of the shell with the array, the reinforcement of the tunnel leads to a decrease in radial displacements and compressive axial stresses ($\sigma_{\eta\eta}$). The effect of the shell on the nature of the change in normal stresses ($\sigma_{\theta\theta}$) is somewhat different: these stresses increase in the central parts of the tunnel arch. As the thickness and stiffness of the sheath material increase, the displacement and stresses decrease. Contact conditions also affect the stresses and permeations of the contour of the section.

2. All the considered load velocities, with a relatively small period T = p / 4 and the fluidity of the medium (0 <A <0.48), the components of the stress-strain state of the earth's surface are practically zero. With a decrease in the period (T/ h <0.4), as calculations have shown, an entire region of the array with zero components begins to form from the earth's surface, which covers the entire array with a sufficiently small period, except for a small thickness of the layer around the tunnel.

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