MATHEMATICAL MODELS OF THE HEAT AND MASS EXCHANGE PROCESS DURING PNEUMO-TRANSPORTATION OF COTTON-RAW

Abstract: In the scientific literature, there is not enough information on the change in the heat-moisture states of raw-cotton and its components during processing. The article defines the patterns of changes in the heat and humidity conditions of raw-cotton and its components during pneumatic transportation, depending on the initial moisture content of raw-cotton and atmospheric air. Mathematical models for calculating the temperature and moisture content of raw cotton during cooling obtained.

Key words: raw-cotton; processing; drying; cooling; humidity; heat transfer; heat capacity; heating.

Introduction

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The effectiveness of the process largely depends on the manufacturability of the processed material. In the primary processing of raw-cotton, its components must have the appropriate thermal and humidity conditions, in which the cleaning and ginning processes are successful, they should be more susceptible to shedding and fiber separation without damage to the seeds, i.e. in technological states [1-5].

In the process of processing, raw-cotton undergoes various effects of thermo-mass transfer phenomena that change the heat of the humidity conditions of raw-cotton and its components. For example, from a riot, raw cotton is conveyed by pneumatic conveying to a drying unit, while it is ventilated with atmospheric air, as a result, heated raw material in riots is cooled in certain values, and a certain amount of free moisture evaporates. Then, during the drying process, it is heated and dried, depending on the initial humidity, the temperature of the drying agent, and the productivity of drying plants on wet cotton according to certain consistent pattern [6, 7] and the next process, are transported by air, and here it is along the way ventilated and cooled.

In the process of cleaning it is also cooled, after which it again goes with the help of pneumatic conveying to the ginning process, cooling down along the way. In the process of ginning, raw-cotton is heated in contact with the surfaces of the saws and the friction between them in a dense raw roller. In the scientific literature, there is not enough information on changes in the heat-moisture states of raw-cotton and...
its components during processing. Therefore, the study of changes in these indicators is relevant from the point of view of the management of technological states in the process of cleaning and ginning.

Objects and methods of research: Given that mainly cotton is transported by air, in the interval from time \( t = 0 \) to time \( t = t^* \), the raw material components are cooled in pneumatic transport under the influence of atmospheric air. Raw-cotton is absorbed from a distance of \( X = L \) or, starting from the cross-section of \( X = 0 \), it is exposed to atmospheric air.

In pneumatic transport, raw cotton moves at a speed of \( V \). Denote the heating temperature of the fiber \( T_1(t) \), the heating temperature of the seeds - \( T_2(t) \), and the air temperature \( T_a \). Imagine that seed cooling occurs through the pulp.

\[
\begin{align*}
\frac{dT_1}{dt} &= \lambda_1 \left( \frac{4(T_{10} - 2T_1 + T_{\text{max}})}{L^2} - c_1 \rho_1 V \frac{T_{\text{max}} - T_{10}}{L} \right) + \alpha_1(T_a - T_1) + \alpha_2[T_2 - T_1] + 0.01 \varepsilon_1 \rho_1 r_{21} \frac{dw_1}{dt} \\
\frac{dT_2}{dt} &= \alpha_2[T_1 - T_2] + 0.01 \varepsilon_2 \rho_2 r_{21} \frac{dw_2}{dt}
\end{align*}
\]

with initial conditions

\[
T_1(0) = T_{10}, \quad T_2(0) = T_{20}
\]

Where \( c_1, c_2 \) - density, respectively, of fiber and seeds; \( \rho_1, \rho_2 \) - density, respectively, of fiber and seeds; \( \lambda_1 \) - coefficient of thermal conductivity of the fiber; \( \alpha_1, \alpha_2 \) - heat transfer coefficients, respectively, of air-fiber and fiber-seed.

We write the system (1) in the form

\[
\begin{align*}
\frac{dT_1}{dt} &= -a_11 T_1 + a_{12} T_2 + f_1(t) \\
\frac{dT_2}{dt} &= -a_{21} T_1 + a_{22} T_2 + f_2(t)
\end{align*}
\]

Where

\[
a_{11} = \frac{\lambda_1 + \alpha_2 \frac{2 \lambda_1}{L^2}}{c_1 \rho_1}, \quad a_{12} = \frac{\alpha_2}{c_1 \rho_1}, \quad a_{11} > a_{12}
\]

\[
a_{21} = \frac{\alpha_2}{c_2 \rho_2}, \quad a_{22} = a_{21}, \quad a_{22} < a_{12}
\]

The heat transfer between the fiber and the seed is characterized as follows:

\[
\alpha_2[T_1(t) - T_2(t)]
\]

Also, the heat exchange between air and fiber has the following expression:

\[
\alpha_1[T_1(t) - T_a(t)]
\]

Where \( T_1(0) \) and \( T_2(0) \) - initial heating temperatures, respectively, of fiber and seeds.

Where \( T_1(0) \) and \( T_2(0) \) - initial heating temperatures, respectively, of fiber and seeds. At any point in the cross-section \([0, L]\), the temperatures of the components \( T_1(t) \) and \( T_2(t) \) are determined by time depending on \( T_a = T_a(t) \) the temperature of the atmospheric air.

Given the foregoing, we write the kinetic equations according to the principle of heat transfer in the following form [8.9]:

To solve system (3), we first consider homogeneous equations.
\[
\begin{align*}
\frac{dT_1}{dt} &= -a_{11}T_1 + a_{12}T_2 \\
\frac{dT_2}{dt} &= -a_{21}T_1 + a_{22}T_2
\end{align*}
\] (4)

For that a characteristic equation is [10,11]
\[
\Delta = \begin{vmatrix}
-a_{11} - \lambda & a_{12} \\
-a_{21} & a_{22} - \lambda
\end{vmatrix} = (-a_{11} - \lambda)(a_{22} - \lambda) + a_{21}a_{12} = 0
\]

And from this we get this one,
\[
\lambda^2 + (a_{11} - a_{22})\lambda + a_{21}a_{12} - a_{11}a_{22} = 0 \quad \text{или} \\
\lambda^2 + (a_{11} - a_{22})\lambda - a_{21}(a_{11} - a_{12}) = 0
\]

The solution to this equation has the form
\[
\lambda_{1,2} = \frac{-a_{11} - a_{22}}{2} \pm \sqrt{\frac{(a_{11} - a_{22})^2}{4} + a_{21}(a_{11} - a_{12})}
\]

As,
\[
a_{11}^2 - 2a_{11}a_{21} + a_{21}^2 + 4a_{22}a_{11} - 4a_{21}a_{12} = a_{11}^2 + 2a_{11}a_{21} + a_{21}^2 - 4a_{21}a_{12} = (a_{11} + a_{21})^2 - 4a_{21}a_{12} = a_{11}^2 + 2a_{21}(a_{11} - 2a_{12}) \geq 4a_{21}a_{11} - 4a_{21}a_{12} = 4a_{21}(a_{11} - a_{12}) > 0
\]

then, the general solution of system (4) can be written in the form
\[
T_1 = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \\
T_2 = \tilde{c}_1 e^{\lambda_1 t} + \tilde{c}_2 e^{\lambda_2 t}
\]

Substituting in (4)
\[
c_1\lambda_1 e^{\lambda_1 t} + c_2\lambda_2 e^{\lambda_2 t} = -a_{11}(c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}) + a_{12}(\tilde{c}_1 e^{\lambda_1 t} + \tilde{c}_2 e^{\lambda_2 t})
\]

Hereof
\[
\begin{align*}
c_1\lambda_1 &= -a_{11}c_1 + a_{12}\tilde{c}_1 \\
c_2\lambda_2 &= -a_{11}c_2 + a_{12}\tilde{c}_2
\end{align*}
\]

Hence,
\[
\begin{align*}
\tilde{c}_1 &= \frac{\lambda_1 + a_{11}}{a_{12}}c_1 \\
\tilde{c}_2 &= \frac{\lambda_2 + a_{11}}{a_{12}}c_2
\end{align*}
\]

Then, the general solution of the homogeneous system can be written as
\[
\begin{align*}
T_1 &= c_1(t)e^{\lambda_1 t} + c_2(t)e^{\lambda_2 t} \\
T_2 &= \frac{\lambda_1 + a_{11}}{a_{12}}c_1(t)e^{\lambda_1 t} + \frac{\lambda_2 + a_{11}}{a_{12}}c_2(t)e^{\lambda_2 t}
\end{align*}
\]

To solve the heterogeneous system (5), we use the method of uncertain coefficients:
\[
\begin{align*}
T_1 &= \frac{\lambda_1 + a_{11}}{a_{12}}c_1(t)e^{\lambda_1 t} + \frac{\lambda_2 + a_{11}}{a_{12}}c_2(t)e^{\lambda_2 t} \\
T_2 &= \frac{\lambda_1 + a_{11}}{a_{12}}c_1(t)e^{\lambda_1 t} + \frac{\lambda_2 + a_{11}}{a_{12}}c_2(t)e^{\lambda_2 t}
\end{align*}
\] (6)

We find the first derivatives

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\( (c_1' + \lambda_1 c_1)e^{\lambda_1 t} + (c_2' + \lambda_2 c_2)e^{\lambda_2 t} = -a_{11}c_1e^{\lambda_1 t} - a_{11}c_2e^{\lambda_2 t} + (\lambda_1 + a_{11})c_1e^{\lambda_1 t} + (\lambda_2 + a_{11})c_2e^{\lambda_2 t} + f_1(t) \)

Then

\[ c_1'e^{\lambda_1 t} + c_2'e^{\lambda_2 t} = f_1(t) \]

\[ \frac{\lambda_1 + a_{11}}{a_{12}} c_1'e^{\lambda_1 t} + \frac{\lambda_2 + a_{11}}{a_{12}} c_2'e^{\lambda_2 t} = f_2(t) \]

Hence,

\[ c_1'e^{\lambda_1 t} + c_2'e^{\lambda_2 t} = f_1(t) \]

\[ \kappa_1 c_1'e^{\lambda_1 t} + \kappa_2 c_2'e^{\lambda_2 t} = f_2(t) \]

Where

\[ c_1(t) = \frac{1}{\kappa_2 - \kappa_1} \int_0^t (\kappa_2 f_1(t) - f_2(t))e^{-\lambda_1 t} dt + c_1(0) \]

\[ c_2(t) = \frac{1}{\kappa_1 - \kappa_2} \int_0^t (\kappa_1 f_1(t) - f_2(t))e^{-\lambda_2 t} dt + c_2(0) \]

(7)

Using the initial conditions of exercise (2) we find

\[ \begin{cases} c_1(0) + c_2(0) = T_{i0} \\ \kappa_1 c_1(0) + \kappa_2 c_2(0) = T_{i0} \end{cases} \]

Solving above, we get

\[ c_1(0) = \frac{\kappa_2 T_{i0} - T_{20}}{\kappa_2 - \kappa_1} \]

\[ c_2(0) = \frac{\kappa_1 T_{i0} - T_{20}}{\kappa_1 - \kappa_2} \]

(8)

Where

\[ \kappa_2 - \kappa_1 = \frac{\lambda_2 + a_{11} - \lambda_1 + a_{11}}{a_{12}} = \frac{\lambda_2 - \lambda_1}{a_{12}} \]

Finally, we obtain formulas (6) - (8) for calculating the temperature of the fiber and seeds during the transportation of raw-cotton.

Problem (1), (2) requires knowledge of the law of change in humidity of raw-cotton and its components. During the cooling period, the “drying” speed usually increases from zero to a value of a certain speed. In this regard, the kinetics of moisture changes in raw-cotton and its components at the initial moment, the cooling period can be approximated by equation [12].

\[ \frac{dW}{d\tau} = k(W_i - W)^m, \quad W|_{\tau=0} = W_H \]

(9)

where \( k \) is the cooling coefficient; \( W_H \) is the initial humidity; \( m \) is a constant that is less than unity and is determined only by the form of the connection of moisture with the material.

Integrating equations (9) we obtain the formula for calculating the cooling duration:

\[ \tau = \frac{1}{k(1-m)}(W_H - W)^{1-m} \]

(10)

The unknown coefficient \( k \) is determined by the least-squares method using the experimental data of humidity \( W_i \) and time \( \tau_i \), from the condition [13]

\[ S = \sum_{i=1}^{N} \left[ \frac{1}{(1-m)}(W_{i0} - W_i)^{1-m} - k\tau_i \right]^2 \rightarrow \text{min} \]

Whence, having equated to zero the first derivative of \( S \) concerning \( k \), we obtain
\[
k = \frac{\sum_{i=1}^{N} Z(W_i) \tau_i}{\sum_{i=1}^{N} \tau_i^2},
\]
\[
Z(W_i) = \frac{1}{(1-m)} (W_{hi} - W_i)^{1-m}
\]
(11)

where \(N\) is the amount of experimental data.

We determine the unknown \(m\) from the condition of reaching the maximum of the pair correlation coefficient by its absolute value:

\[
\max |R|
\]

where

\[
R = \frac{\sum_{i=1}^{N} Z(W_i) \tau_i}{\sqrt{\sum_{i=1}^{N} Z^2(W_i) \sum_{i=1}^{N} \tau_i^2}}
\]

To calculate the change in humidity from (10) we get

\[
W = W_{hi} - (k \cdot (1-m) \cdot \tau)^{1-m}
\]
(12)

Where the cooling rate is calculated by the formula:

\[
dW \over d\tau = -k^{1-m} \cdot \tau^{1-m} (1-m)^{1-m}
\]
(13)

**Results and discussion:** Figure 1 and fig. 2 shows the curves of changes in the temperature of the fiber and seeds. The parameters entering into equations (1) - (2) selected from the literature \[14, 15\]. To verify the adequacy of the model and its solution, experimental data used.

The calculations carried out with the following values: the initial temperature of the fiber 71°C and seeds 57°C (Fig. 1) and, respectively, 40°C and 35°C (Fig. 2), air temperature \(T_{\text{air}} = 10°C\).

The parameters entering into equations (1) - (2) selected from the literature \[14, 15\]. To verify the adequacy of the model and its solution, experimental data used.

In the Uchkurgan ginnyery, raw cotton of selection grade S-6524, manual picking, industrial-grade I, with an initial moisture content of W = 10.0 and industrial-grade II, with an initial moisture content of 14.3% was studied. After drying them in a 2SB-10 dryer at a temperature of drying agent \(T = 100\) and 200 °C, the productivity of 3.5 and 10 t / h and after pneumatic transportation at a distance of 25 m, a change in humidity and heating temperature of raw-cotton and its components determined.
As can be seen from fig. 1 and fig. 2, at the beginning of the cooling process, the heating temperature of the seeds changes slightly, and subsequently, the cooling intensity increases, i.e. the seeds cool quickly. At the end of the process, the rate of cooling of the seeds decreases. This is because, at the beginning of the process, the temperature of the fiber is higher than that of the seeds.

A comparison of the experimental data on the temperature changes of raw cotton components during pneumatic conveying and the calculated ones according to the solution (6) - (8) show that the relative error is no more than 5%, which allows the use of this algorithm to calculate the temperature and moisture content of raw cotton during cooling.

References:


