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ON THE EFFECT OF HARMONIC WAVES ON AN ORTHOTROPIC CYLINDRICAL SHELL IN A VISCOELASTIC MEDIUM

Abstract: In this paper, we consider the dynamic reaction of an orthotropic cylindrical shell (pipe) in a viscoelastic medium that is subjected to seismic action. The cylindrical shell is infinitely long, therefore, the problem of a plane deformable state is considered. It is assumed that the shell is completely connected to an environment of infinite extent. Only axisymmetric and non-axisymmetric oscillations with an incident compression wave (transverse or shear) were studied. The results were obtained for different characteristics of viscoelastic media - hard, medium and soft. It was found that the orthotropic parameters do not affect the reaction of the cylindrical shell. The degree of influence of orthotropens strongly depends on the degree of rigidity of the environment.

Key words: wave, dynamic reaction, viscoelastic medium, seismic impact, shell.

Language: English

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Introduction

Setting the task. An infinitely long, homogeneous, isotropically deformable cylinder with an ideal compressible liquid, located in an infinite viscoelastic medium, is affected by a harmonic expansion wave (transverse or shear) (Fig.1). The front of the harmonic (or seismic) waves can be parallel to the axis of the cylinder. Here are the outer and inner radii of the cylindrical layer. The main purpose of the work is to determine the stress-strain state of the

cylindrical layer and the environment under the influence of longitudinal (or transverse) harmonic waves. Under the assumption of a generalized deformed state, the equation of motion in displacements has the form [1]

$$(\tilde{\lambda}_j + 2\tilde{\mu}_j) \text{grad div} \tilde{u}_j - \tilde{\mu}_j \text{rot rot} \tilde{u}_j + \tilde{b}_j = \rho_j \frac{\partial^2 \tilde{u}_j}{\partial t^2}, \quad (1)$$

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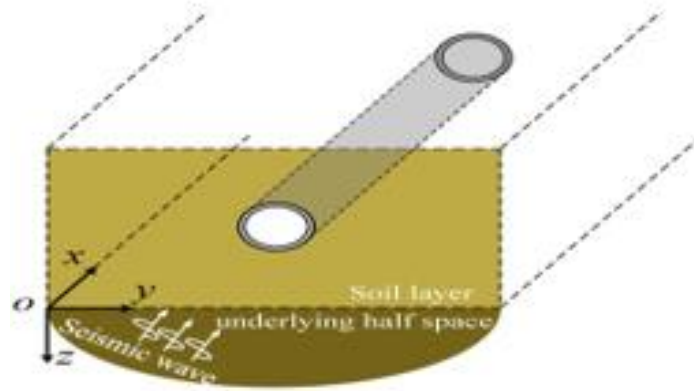


Fig.1. Calculation scheme.

where λ_j and μ_j (in the index, $j=1, 2$, $j=1$ - refer to the environment, $j=2$ - to the layer) are the operator modulus of elasticity

$$\tilde{\lambda}_j f(t) = \lambda_{0j} \left[f(t) - \int_{-\infty}^t R_{\lambda}^{(i)}(t-\tau) f(\tau) d\tau \right],$$

$$\tilde{\mu}_j f(t) = \mu_{0j} \left[f(t) - \int_{-\infty}^t R_{\mu}^{(i)}(t-\tau) f(\tau) d\tau \right]$$

$\vec{b}_j$ - vector of density of volumetric forces ($b_j=0$);
 $f(t)$ - some function; ρ_j - density of materials,
 $R_{\mu}^{(i)}(t-\tau)$ and $R_{\lambda}^{(i)}(t-\tau)$ - relaxation core,
 λ_{0j}, μ_{0j} - instantaneous elastic modulus of a viscoelastic material, $\vec{u}_j(u_{rj}, u_{\theta j})$ - displacement vector, which depends on r, θ, t . For velocity potentials of liquid particles, at pressures up to 100 MPa, motion in the liquid is satisfactorily described by wave motion. [2]

$$\Delta \varphi_0 = \frac{1}{C_o^2} \frac{\partial^2 \varphi_0}{\partial t^2}, \quad (2)$$

where C_o is the acoustic speed of sound in the liquid. The potential and φ_0 the fluid velocity vector are $\vec{V} = \text{grad} \varphi_0$ related by the dependence. The fluid pressure is $r=R_0$ determined using the linearized

Cauchy-Lagrange integral - the fluid pressure on the wall of the cylindrical layer and ρ_o - the density of the fluid. Under the condition of continuous flow of liquid, the normal component of the velocity of the liquid and the layer on the surface of their contact $r=R_0$ must be equal

$$\left. \frac{\partial \varphi_0}{\partial r} \right|_{r=R_0} = \left. \frac{\partial u_{r2}}{\partial t} \right|_{r=R_0}, \quad (3)$$

where are the displacements of the layer along u_{r2} the normal. At the contact of two bodies $r=R$, the displacements and stresses are equal (condition of rigid contact)

$$u_{r1} = u_{r2}; \sigma_{rr1} = \sigma_{rr2}; u_{\theta 1} = u_{\theta 2}; u_{z1} = u_{z2};$$

$$\sigma_{r\theta 1} = \sigma_{r\theta 2}; \sigma_{rz1} = \sigma_{rz2} \quad (4, a)$$

Note that in the case of sliding contact of the soil along the surface of the pipe, the last equation in (4) will take the form [3]:

$$\sigma_{r\theta 1} = 0, \sigma_{rz1} = 0, \sigma_{r\theta 2} = 0, \sigma_{rz2} = 0. \quad (4, b)$$

The boundary conditions free from load are set in the form:

$$r=a; \sigma_{rr}=0; \sigma_{r\theta}=0; \sigma_{rz}=0, \quad (4, v)$$

where are normal $\sigma_{r\theta}$ and σ_{rz} are shear stresses. The corresponding stresses and displacements surrounding the cylindrical pipes in the displacement potentials have the following form [4,5]:

$$\sigma_{rr} = \lambda \nabla^2 \varphi + 2\mu \left[\left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{\partial}{\partial r} \left(\gamma \frac{\partial \varphi}{\partial \theta} \right) + e \frac{\partial^3 \chi}{\partial r^2 \partial x} \right) \right];$$

$$u_r = \frac{\partial \varphi}{\partial r} + \frac{1}{r} \frac{\partial \varphi}{\partial \theta} + e \frac{\partial^2 \chi}{\partial r \partial \theta}, u_{\theta} = \frac{1}{r} \frac{\partial \varphi}{\partial \theta} - \frac{\partial \varphi}{\partial r} + e \frac{1}{r} \frac{\partial \chi}{\partial \theta \partial x}$$

$$u_r = \frac{\partial \varphi}{\partial r} - e \left[\frac{1}{\gamma} \frac{\partial}{\partial \gamma} \left(\gamma \frac{\partial \chi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \chi}{\partial \theta^2} \right] \quad (5)$$

$$\sigma_{xx} = \lambda \nabla^2 \varphi + 2\mu \left[\left(\frac{\partial^2 \varphi}{\partial x^2} - e \left(\frac{\partial}{\partial x} \nabla^2 \chi - \frac{\partial^3 \chi}{\partial x^3} \right) \right) \right]$$

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$$\begin{aligned}\sigma_{\theta\theta} &= \lambda \nabla^2 \varphi + 2\mu \left[\frac{1}{r} \left(\frac{\partial}{\partial r} + \frac{1}{r} \frac{\partial^2 \varphi}{\partial r^2} \right) + \frac{1}{r} \left(\frac{1}{r} \frac{\partial \varphi}{\partial \theta} - \frac{\partial^2 \varphi}{\partial r \partial \theta} \right) + \frac{e}{r} \left(\frac{\partial^2 \chi}{\partial r \partial \chi} + \frac{1}{r} \frac{\partial^3 \chi}{\partial e^2 \partial \chi} \right) \right] \\ \sigma_{r\theta} &= \mu \left[2 \left(\frac{1}{r} \frac{\partial^2 \varphi}{\partial \theta \partial \chi} - \frac{1}{r^2} \frac{\partial \varphi}{\partial \theta} \right) + \left(\frac{1}{r^2} \frac{\partial^2 \varphi}{\partial \theta^2} - r \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \varphi}{\partial r} \right) \right) \right] + 2e \left(\frac{1}{r} \frac{\partial^3 \chi}{\partial r \partial \theta \partial \chi} \right) \\ \sigma_{rx} &= \mu \left\{ 2 \frac{\partial \varphi}{\partial r \partial x} + \frac{1}{r} \frac{\partial^2 \varphi}{\partial \theta \partial x} + e \left[\frac{\partial^3 \chi}{\partial \gamma \partial x} - \frac{\partial}{\partial r} (\nabla^2 \chi) \right] \right\}; \\ \sigma_{\theta x} &= \mu \left\{ \frac{2}{r} \frac{\partial^2 \varphi}{\partial \theta \partial x} - \frac{\partial^2 \varphi}{\partial r \partial x} + e \left[\frac{2}{r} \frac{\partial^2 \chi}{\partial \theta \partial x^2} - \frac{1}{r} \frac{\partial}{\partial \theta} (\nabla^2 \chi) \right] \right\}; \\ \nabla^2 f &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial x^2},\end{aligned}$$

We accept the integral terms in (1) as small, then the function, where is a slowly varying function $\varphi(t) = \psi(t)e^{-i\omega_R t}$ of time, ω_R is a real constant. Next, using the freezing procedure [6], we replace relations (2) with approximate ones of the form

$$\begin{aligned}\bar{\lambda} \varphi &= \lambda_{01} [1 - \Gamma_\lambda^C(\omega_R) - i\Gamma_\lambda^S(\omega_R)]; \\ \bar{\mu} \varphi &= \mu_{01} [1 - \Gamma_\mu^C(\omega_R) - i\Gamma_\mu^S(\omega_R)]\varphi,\end{aligned}\quad (6)$$

where

$$\begin{aligned}\Gamma_\lambda^C(\omega_R) &= \int_0^\infty R_\lambda(\tau) \cos \omega_R \tau d\tau; \quad \Gamma_\lambda^S(\omega_R) = \int_0^\infty R_\lambda(\tau) \sin \omega_R \tau d\tau, \\ \Gamma_\mu^C(\omega_R) &= \int_0^\infty R_\mu(\tau) \cos \omega_R \tau d\tau; \quad \Gamma_\mu^S(\omega_R) = \int_0^\infty R_\mu(\tau) \sin \omega_R \tau d\tau,\end{aligned}$$

- respectively cosine and sine Fourier images of the relaxation kernel of the material. We represent the vector of environmental displacements in the form of a potential

$$\vec{u} = \text{grad } \phi_\kappa + \text{rot } \vec{\psi}_\kappa, \text{div } \vec{\psi}_\kappa = 0,$$

At each point, the components of the displacement vector of the surface of the cylindrical cavity are expressed in terms of longitudinal (φ) and transverse potentials ($\psi_r, \psi_\theta, \psi_z$), which satisfy the following partial differential equations:

$$\begin{aligned}\nabla^2 \phi - \frac{1}{c_p^2 \Gamma_\kappa} \frac{\partial^2 \phi}{\partial t^2} &= 0; \\ \nabla^2 \psi_z - \frac{1}{c_s^2 \Gamma_\kappa} \frac{\partial^2 \psi_z}{\partial t^2} &= 0; \\ \nabla^2 \psi_\theta - \frac{\psi_\theta}{r^2} + \frac{2}{r^2} \frac{\partial \psi_r}{\partial \theta} - \frac{1}{c_s^2 \Gamma_\kappa} \frac{\partial^2 \psi_\theta}{\partial t^2} &= 0; \\ \nabla^2 \psi_r - \frac{\psi_r}{r^2} - \frac{2}{r^2} \frac{\partial \psi_\theta}{\partial \theta} - \frac{1}{c_s^2 \Gamma_\kappa} \frac{\partial^2 \psi_r}{\partial t^2} &= 0. \\ u_r &= \frac{\partial \phi}{\partial r} + \frac{1}{r} \frac{\partial \psi_z}{\partial \theta} - \frac{\partial \psi_\theta}{\partial z}; \\ u_\theta &= \frac{1}{r} \frac{\partial \phi}{\partial \theta} + \frac{\partial \psi_r}{\partial z} - \frac{\partial \psi_z}{\partial r}; \\ u_z &= \frac{\partial \phi}{\partial z} + \frac{\partial \psi_\theta}{\partial r} + \frac{\psi_\theta}{r} - \frac{1}{r} \frac{\partial \psi_r}{\partial \theta}.\end{aligned}\quad (7)$$

$$\text{Here } c_p^2 = (\lambda_{01} + 2\mu_{01}) / \rho; \quad c_s^2 = \mu_{01} / \rho,$$

$$\Gamma_\kappa = 1 - \Gamma^C - i\Gamma^S, \quad \Gamma_\lambda^C = \Gamma_\mu^C = \Gamma^C, \quad \Gamma_\lambda^S = \Gamma_\mu^S = \Gamma^S, \quad R_\lambda = R_\mu = R$$

The solution to equation (7) is sought in the form:

$$\begin{aligned}\varphi(r, \theta, z, t) &= \sum_{n=0}^\infty \varphi_n(\alpha r) \begin{Bmatrix} \cos n\theta \\ -\sin n\theta \end{Bmatrix} e^{\pm i\gamma_p z} e^{-i\alpha t}; \\ \psi_r(r, \theta, z, t) &= \sum_{n=0}^\infty \psi_{nr}(\beta r) \begin{Bmatrix} \sin n\theta \\ -\cos n\theta \end{Bmatrix} e^{\pm i\gamma_p z} e^{-i\alpha t}; \\ \psi_\theta(r, \theta, z, t) &= \sum_{n=0}^\infty \psi_{n\theta}(\beta r) \begin{Bmatrix} \cos n\theta \\ -\sin n\theta \end{Bmatrix} e^{\pm i\gamma_p z} e^{-i\alpha t}; \\ \psi_z(r, \theta, z, t) &= \sum_{n=0}^\infty \psi_{nz}(\beta r) \begin{Bmatrix} \sin n\theta \\ \cos n\theta \end{Bmatrix} e^{\pm i\gamma_p z} e^{-i\alpha t},\end{aligned}\quad (8)$$

where n is an integer; γ_p - dimensionless propagation constant; ω - complex frequency; $r = \frac{r_1}{a_0}, z = \frac{z_1}{a_0}$.

Substituting (8) into (7), we obtain the following ordinary differential equations:

$$\begin{aligned}\frac{d^2 \varphi_n}{dr^2} + \frac{1}{r} \frac{d\varphi_n}{dr} + \left(\alpha^2 - \frac{n^2}{r^2} \right) \varphi_n &= 0; \\ \frac{d^2 \psi_{zn}}{dr^2} + \frac{1}{r} \frac{d\psi_{zn}}{dr} + \left(\beta^2 - \frac{n^2}{r^2} \right) \psi_{zn} &= 0;\end{aligned}\quad (9)$$

$$\begin{aligned}\frac{d^2 \psi_{\theta n}}{dr^2} + \frac{1}{r} \frac{d\psi_{\theta n}}{dr} + \frac{1}{r^2} (-n^2 \psi_{\theta n} + 2n\psi_{\theta n} - \psi_{\theta n}) + \beta^2 \psi_{\theta n} &= 0; \\ \frac{d^2 \psi_{rn}}{dr^2} + \frac{1}{r} \frac{d\psi_{rn}}{dr} + \frac{1}{r^2} (-n^2 \psi_{rn} + 2n\psi_{\theta n} - \psi_{rn}) + \beta^2 \psi_{rn} &= 0;\end{aligned}$$

Here are indicated:

$$\alpha^2 = \frac{\Omega^2}{\gamma^2} - \gamma_p^2; \quad \beta^2 = \Omega^2 / \Gamma_\kappa - \gamma_p^2; \quad \Omega = \frac{\omega \alpha}{c_s}; \quad \gamma^2 = \frac{2(1-\nu)}{1-2\nu} \Gamma_\kappa;$$

$$\Gamma^S = \frac{A\Gamma(\alpha)}{(\omega^2 + \beta^2)^{3/2}} \text{Sin}(\alpha \arctg \frac{\omega}{\beta}),$$

$$\Gamma^C = \frac{A\Gamma(\alpha)}{(\omega^2 + \beta^2)^{3/2}} \text{Cos}(\alpha \arctg \frac{\omega}{\beta}).$$

At infinity, the potentials of longitudinal and transverse waves as $r \rightarrow \infty$ satisfy the Sommerfeld

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radiation condition [7,8]. The first two equations in (7) obviously have the following solutions for the case of a cylindrical cavity:

$$\begin{aligned}\varphi_n(r) &= F_n H_n^{(1)}(\alpha r), \\ \psi_{zn}(r) &= M_{1n} H_n^{(1)}(\beta r).\end{aligned}\quad (10)$$

Here $H_n^{(1)}(\beta_k r)$ is the Hankel function of the n th kind, 1st order. To construct solutions to the remaining two equations in (7), it is advisable to consider their sum and difference. In this case, two equations are obtained, the solutions of which are expressed through Hankel functions of different orders

$$\begin{aligned}u_r &= \sum_{n=0}^{\infty} \left[F_n \frac{dH_n^{(1)}(\alpha r)}{dr} + D_{2n} i \gamma_p H_{n+1}^{(1)}(\beta r) + M_{1n} n H_n^{(1)}(\beta r) / r \right] \begin{pmatrix} \cos n\theta \\ -\sin n\theta \end{pmatrix} e^{i(-\alpha r + \gamma_p z)}, \\ u_\theta &= \sum_{n=0}^{\infty} \left[-F_n n H_n^{(1)}(\alpha r) / r + D_{2n} i \gamma_p H_{n+1}^{(1)}(\beta r) - M_{1n} \frac{dH_n^{(1)}(\beta r)}{dr} \right] \begin{pmatrix} \sin n\theta \\ \cos n\theta \end{pmatrix} e^{i(-\alpha r + \gamma_p z)}, \\ u_z &= \sum_{n=0}^{\infty} \left[-F_n i \gamma_p H_{n+1}^{(1)}(\alpha r) - D_{2n} \left[\frac{dH_{n+1}^{(1)}(\beta r)}{dr} + \frac{n+1}{r} H_{n+1}^{(1)}(\beta r) \right] \right] \begin{pmatrix} \cos n\theta \\ -\sin n\theta \end{pmatrix} e^{i(-\alpha r + \gamma_p z)}.\end{aligned}\quad (11)$$

Expressions (11) contain three arbitrary constants F_n, D_{2n}, M_{1n} and satisfy Ω the equation of motion for arbitrary values of frequency. In the case of a stress-free cylindrical surface and when the normal ($\sigma_{rr} = 0$) and tangential stresses ($\sigma_{r\theta} = \sigma_{rz} = 0$) on it are equal to zero, the dispersion equation has the form:

$$\|c_{ij}\| = 0, \quad i, j = 0, 1, 2, 3. \quad (12)$$

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$$\begin{aligned}c_{11} &= (n^2 - 1 - \Omega^2 + \gamma_p^2) H_n^{(1)}(\alpha), \\ c_{12} &= (n^2 - 1 - \beta^2) H_n^{(1)}(\beta), \\ c_{13} &= 2(n^2 - 1) [\beta H_{n-1}^{(1)}(\beta) - n H_n^{(1)}(\beta)] - \beta^2 H_n^{(1)}(\beta), \\ c_{21} &= \alpha H_{n-1}^{(1)}(\alpha) - (n+1) H_n^{(1)}(\alpha), \\ c_{22} &= \beta H_{n-1}^{(1)}(\beta) - (n+1) H_n^{(1)}(\beta), \\ c_{23} &= (2n^2 + n - \gamma_p^2) H_n^{(1)}(\beta) - 2\beta H_{n-1}^{(1)}(\beta), \\ c_{31} &= \alpha H_{n-1}^{(1)}(\alpha) - n H_n^{(1)}(\alpha), \\ c_{32} &= (1 - \Omega^2 / 2\gamma_p^2) (H_{n-1}^{(1)}(\beta) - n H_n^{(1)}(\beta)), \\ c_{33} &= n^2 H_n^{(1)}(\beta).\end{aligned}$$

Equation (9) for each value $n = 1, 2, 3$, determines the relationship between the dimensionless frequency γ_p and Ω the dimensionless propagation constant at ν - Poisson's ratio. The most effective way to solve equation (12) is the Muller method [9,10], which was used here. Without revealing the frequency to determine, but by calculating at each step only Ω its

$$\begin{aligned}\psi_{rn}(r) &= D_{1n} H_{n-1}^{(1)}(\beta r) + D_{2n} H_{n+1}^{(2)}(\beta r), \\ \psi_{\theta n}(r) &= D_{1n} H_{n-1}^{(1)}(\beta r) - D_{2n} H_{n+1}^{(2)}(\beta r).\end{aligned}$$

After finding the functions of the radial coordinate, representations (10) will contain four arbitrary constants that have a certain redundancy. In what follows it is assumed: $D_{1n} = 0$, then $\psi_{rn} = -\psi_{\theta n}$. This is convenient from the point of view of subsequent satisfaction of boundary conditions on a cylindrical surface. Thus, the components of the displacement vector in the vicinity of the cylindrical cavity are represented as:

value for a fixed value, the indicated method can be used to find $\Omega = \Omega_R + i\Omega_I$ the natural complex frequencies. Damping coefficients allow us to judge the damping properties of the system under consideration. In technology, to estimate the rate of attenuation of oscillatory processes, another characteristic is used, namely, the logarithmic decrement of oscillation attenuation. It is related to the damping coefficient by the following formula:

$$\delta = 2\pi\Omega_I / \Omega_R.$$

All expressions for stresses and displacements have the form:

$$(\mathbf{Re} + i \mathbf{Im}) e^{-i\alpha t} = (\mathbf{Re}^2 + \mathbf{Im}^2)^{1/2} e^{-i(\alpha t - \gamma)}.$$

As you can see, the solution to the problem is expressed through special Bessel and Hankel functions of the 1st and 2nd kind. As their argument increases, series (8) converges. Therefore, based on numerical experiments, it was established that out of 5-6 terms of the series, the accuracy reached $10^{-8} - 10^{-6}$. As the relaxation kernel of a viscoelastic material, we take the three-parameter kernel

$$R(t) = \frac{A e^{-\beta t}}{t^{1-\alpha}} \quad (13)$$

Rzhanitsyn - Koltunov [11,12], which has a weak singularity, where A, α, β are the parameters materials. Equation (12) is $n \geq 0$ valid for all integers. Each value determines the properties of natural waves. For example, we have a family $n=1$ of bending natural waves. When $n=0$ the dispersion equation is reduced to the product of two factors of the element of the second row of the third column. The first of these factors gives the dispersion equation for the family of

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torsional natural waves. The second factor gives the dispersion equation for the family of longitudinal natural waves on the surface of the cavity.

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