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SOME INFORMATION FROM GRAPHE THEORY AND ITS APPLICATION TO THE STUDY OF SPARKLY SYMMETRIC MATRICES

Abstract: *the basic concepts of graph theory are presented and their correspondence with matrix concepts is established. It is known that few results of graph theory have found direct application in the analysis of sparse matrix computations and its notations, the concepts are convenient and useful in describing algorithms, as well as in recognizing and characterizing the structure of a matrix.*
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Introduction

In this paper, the basic concepts of graph theory are presented and their correspondence with matrix concepts is established. It is known that few results of graph theory have found direct application in the analysis of sparse matrix computations and its notations, the concepts are convenient and useful in describing algorithms, as well as in recognizing and characterizing the structure of a matrix. However, when using graph theory in such analysis it is easy to cross the reasonable line; the result will often be that the terminological elegance obscures the meaning of some essentially simple ideas. Therefore, perhaps sacrificing uniformity of presentation in the paper, definitions and results are given in terms of both graph theory and matrix theory. For the same reasons, most of the terms of graph theory are introduced where there is a need for them.

1. Basic terminology and some definitions

In this paper, a graph G=(X,E) is represented as consisting of a finite set of nodes, or vertices, together with a set E of edges, which are unordered pairs of vertices. An ordering (labeling) of a graph G=(X,E) is simply a mapping of the set {1,2,…,N} onto X; here N is the number of nodes in G. Unless otherwise specified, a graph is considered unordered. A graph G labeled by will be denoted by $G^{\alpha} = (X^{\alpha}, E)$.

By introducing graphs, one can facilitate the study of sparse matrices; therefore, connections are established between graphs and matrices. Let A be a symmetric matrix of order N.

An ordered (labeled) graph of a matrix A, denoted G^A=(X^A,E^A), is a graph for which the N vertices of G^A are numbered from 1 to N {x_i,x_j}∈E^A if and only if a_ij=a_ji≠0,i≠j. Here x_i is the node of X^A with label i. Fig. 1 illustrates the structure of the matrix and its labeled graph. To emphasize the correspondence of the i-th diagonal element of the matrix with the node i in a circle. Offdiagonal non-zero elements are denoted by the symbol *.

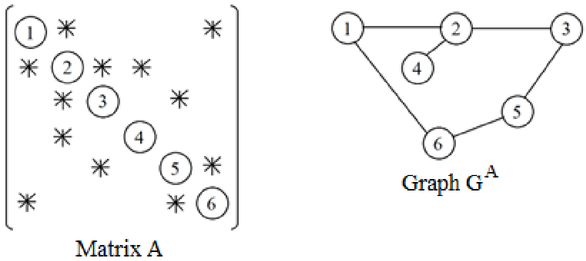

Figure 1. - Matrix and its labeled graph. The symbol * denotes a non-zero element of A.

If PI is an arbitrary ()-permutation matrix, then the unlabeled graphs of the matrices A and PART coincide, and coincide, but the corresponding labelings are different. Thus, the unlabeled graph A represents the structure of A without mentioning the equivalence of the matrices PART , where P is any ()-permutation matrix. Finding a “good” permutation for A can be viewed as finding a good labeling for its graph. Fig. 2 illustrates this.

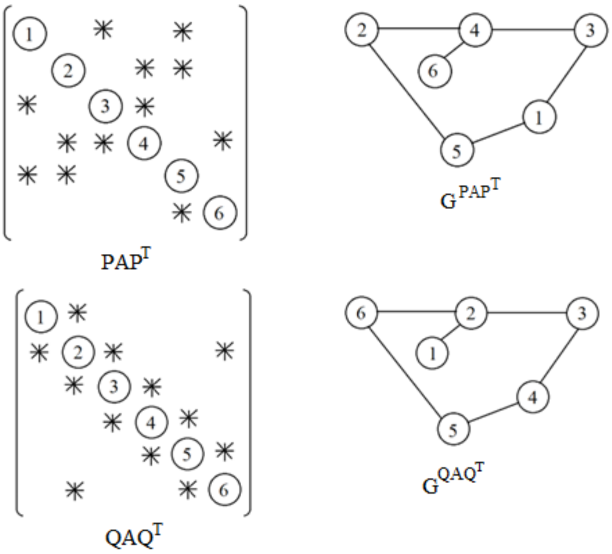

Figure 2. - The graph of Figure 1 with different labelings and the structures of the corresponding matrices. Here P and Q denote the permutation matrices.

Some definitions of graph theory are related to unlabeled graphs. To interpret such definitions in matrix terms, one must introduce the corresponding matrix, which immediately leads to an ordering of the graph. However, one should assign some meaning to the specific ordering chosen in matrix examples and interpretations. When one speaks of "the matrix corresponding to a graph G", one either specifies some ordering for G, or implies that the ordering may be arbitrary.

Two nodes x and y in G are called adjacent if {x,y}∈ E. If Y⊂X1, then the adjacent set for Y is Adj(Y) = {x∈X – Y | {x,y}∈ E for some y∈Y }

In words: Adj(Y) is the set of nodes in G that do not belong to Y but are adjacent to at least one node in Y. Figure 3 illustrates the matrix interpretation of the set Adj(Y). For convenience, the nodes of Y have been labeled with consecutive integers. If Y has a single node y, one should write Adj(y) instead of the formally correct Adj({y}).

For YX, the degree of Y, denoted by Deg(Y), is the number |Adj(y)|, where |S| denotes the number of elements of the set S. Again, if Y has a single vertex y, we will write Deg(y), not Deg({y}). For example, in Fig. 3, Deg(x_2)=3.

A part G’ = (X’,E’) of a graph G is a graph for which X’⊂X and E’⊂E. If Y⊂X, then the subgraph G(Y) is a part (Y,E(Y)) of the graph G such that $E(Y) = \{\{x,y\} \in E | x \in Y, y \in Y\}$

In matrix language, a subgraph of G(Y) is the graph of the matrix obtained from the matrix of graph G by deleting all rows and columns except those corresponding to Y. This is illustrated in Figure 4.

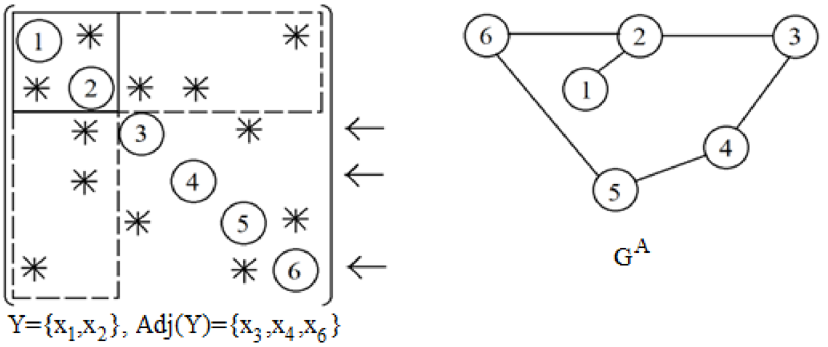

Figure 3. - Illustration of the concept of a contiguous set for the set YX

A subgraph is called a clique if all its nodes are pairwise adjacent. In matrix terminology, a clique corresponds to a filled submatrix. For example, G({x_1, x_4}) is a clique.

The example in Fig. 4 illustrates the concept, namely, connectivity of a graph. If x and y are distinct nodes of G, then a path from x to y of length l 1 is an ordered set of *l+1* distinct nodes (v_1,v_2,…,v_{l+1}) such that v_i+*1*Adj(v_i), *i*=1,2,…,*l*, and v_1=x, v_{l+1}=y. A graph is called connected if each pair of distinct nodes is connected by at least one path. Otherwise, G is disconnected and consists of two or more connected components. If we talk about matrix analogies, it should be clear that for

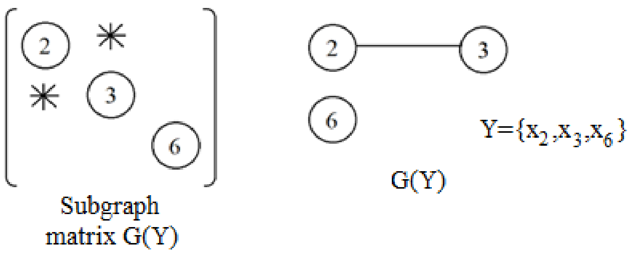

Figure 4. - An example of a subgraph G(Y) and the corresponding submatrix. The original graph G is shown in Fig. 1.

For an unconnected graph G consisting of k connected components, when these components are successively labeled, the corresponding matrix will be block-diagonal, and each diagonal block is associated with a connected component. The graph G(Y) in Fig. 4 is ordered in this way, and the corresponding matrix is block-diagonal.

Fig. 5 shows a path in the graph and its matrix interpretation. Finally, a set is called a separator of a connected graph G if the subgraph G(X-Y) is disconnected. For example, Y={x_3,x_4,x_5} is a separator of the graph in Fig. 5, because the graph G(X-Y) consists of three components with node sets {x_1},{x_2} and {x_6,x_7}, respectively.

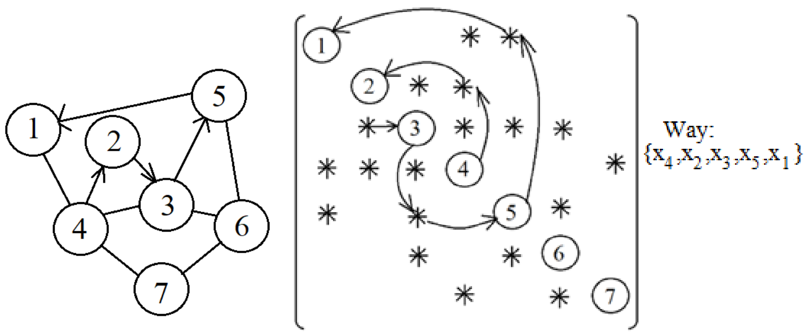

Figure 5. - Path in the graph and the corresponding matrix interpretation

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