SOI: 1.1/TAS DOI: 10.15863/TAS
International Scientific Journal
Theoretical & Applied Science

p-ISSN: 2308-4944 (print) **e-ISSN:** 2409-0085 (online)

Year: 2024 **Issue:** 12 **Volume:** 140

Published: 17.12.2024 **http://T-Science.org**

Issue Article

C.T. Mukhambetzhanov
U.Zholdasbekov Institute of Mechanics and Mechanical Engineering
Doctor of Physical and Mathematical Sciences
saltanbek.kaznu@gmail.com

A. Zheksenkhan
M.Kh.Dulaty Taraz University
Master student
zeksenhana@gmail.com

E.A. Zhakashova
M.Kh.Dulaty Taraz University
senior lecturer
ela.jakashova.80@mail.ru

MATHEMATICAL MODEL OF BODY MOVEMENTS UNDER THE DYNAMIC INFLUENCE OF MOVING LOADS

Abstract: *The construction of mathematical models of the movement of elastic coupling installations of mobile loads and their solution by analytical and modern calculation methods, the construction of programs based on the use of computational methods in research.*

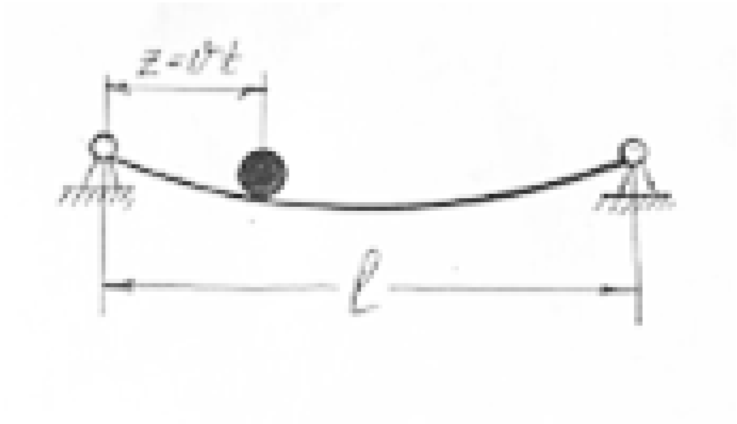
Key words: *dynamic effect, moving loads, elastic devices, deviation, bending of the medium, critical speeds.*
Language: *English*
Citation: Mukhambetzhanov, C.T., Zheksenkhan, A., & Zhakashova, E.A. (2024). Mathematical model of body movements under the dynamic influence of moving loads. *ISJ Theoretical & Applied Science, 12 (140),* 133-138.
Soi: https://s-o-i.org/1.1/TAS-12-140-23 ***Doi***: crossref https://dx.doi.org/10.15863/TAS.2024.12.140.23
Scopus ASCC: *2600.*

Introduction

Scholars began to deal with such a problem in the 19th century. The reason for this is the catastrophe that happened in 1847 on the Chester Bridge in England, that is, the bridge burst while moving in a straight line through the suspension bridge. It was explained by the effect of the resonance phenomenon caused by the equalization of the vibration frequency of the suspension bridge with the frequency of the soldiers' feet. It became clear that such a problem is relevant when comparing the effect of moving loads acting on elastic devices with the effect of forces in stationary, that is, statistical conditions. [1]

Scientists who studied from a theoretical point of view assumed that the dynamic effect is almost twice as much as the statistical effect. English scientist G. Cox, who made such a conclusion, explains it in the following way.

Let a load of weight P act on a fixed beam, moving along the beam. The stiffness of the cross section of the beam during bending is EY, the length l and the deflection of the beam through the center of the beam is f (1 – picture)



Picture 1. Bending of a beam under the influence of a single load

Then, in this case, the force P does work equal to Pf, and also the potential energy of the bending of the beam at this time $48EIf^2/2l^3$ is equal.

By equating energy with work, we find the deflection f of the beam.

$$pf = 48EIf^2/2l^3,$$
$$pl^3 = 24EIf,$$
$$f = pl^3/24EI$$

Dynamic deviation $f_{дин} = pl^3/24EI$ statistical $f_{ст} = pl^3/48EI$ we see that it is twice as much as the deviation. Here we do not take into account the mass of the beam, and if we take into account, then we should take into account its kinetic energy. G. Stokes said that this conclusion of Cox was incorrect and gave his own concept. He proved the error of Cox's energies in his child, that is, he proved that he did not take into account the work of the horizontal force, which ensures the constant speed of the load moving along the beam. If we assume that there is no such horizontal force, the speed of a body moving along a curved trajectory cannot be constant, so the change in the kinetic energy of the body should be taken into account. Since then, many scientific works and studies have been conducted on the effects of moving loads on bridges.

According to the scheme of influence of inertial properties of bodies, the question of the influence of moving loads will be the following four cases. These conditions for a two-span beam can be seen in the following table. [4]

Table 1.

Variants of the problem	Consider mass	
	beam	cargo
	no	no
	no	exists
	exists	no
	exists	exists

Now let's briefly describe these situations.

1 case. All inertial effects are assumed to be negligible and the pressure of the load on the beam is assumed to be equal to the weight of the load.

In this case, the non-dynamic kinematics of the load is taken into account, that is, the variable distance Z with time ($z = \upsilon t$). At the same time, it is necessary to build special graphs to solve such problems. These graphs are graphs of changes in internal effects or displacements of load coordinates. When constructing graphs, the weight of the load is equal to one.

Case 2. In this case, the mass of the elastic beam is taken into account, but the load pressure N is a statistical variation. Since the direction of deflection is downward, the load pressure N on the beam is expressed as

$$N = mg - \frac{md^2y}{dt^2} = mg - \frac{md^2y}{dz^2} * \upsilon^2 \qquad (1)$$

(1) we know the displacement of the load from the variable y in the expression or the deflection of the beam caused by the load. The second term of this expression should be understood as the vertical inertial force of the load.[2]

The concept of this type of calculation was first proposed by Professor F. Willis of Cambridge University. According to his conclusion, he believed that the beam deviates only under the influence of force N. According to his calculations, it was assumed that the deviation of the beam under the influence of the load is calculated by the following equation, i.e

$$y = Nz^2(l-z)^2/3lEI$$

As we can see from this expression, although it appears only as a static solution, the magnitude of the force N included in the equation is not known in advance, and according to equation (1), we can obtain the differential dependence of the following y deviation by deriving the force N from the equations y

$$y = \frac{\left(mg - \frac{md^2y}{dz^2} \cdot \upsilon^2\right) \cdot Z^2(l-z)^2}{3lEI}$$

$$3lEIy = mg \cdot Z^2(l-z)^2 - m\upsilon^2 \cdot Z^2(l-z)^2 \frac{d^2y}{dz^2}$$

$$\frac{d^2y}{dz^2} = \frac{mgz^2(l-z)^2}{m\upsilon^2 Z^2(l-z)^2} - \frac{3lEIy}{m\upsilon^2 z^2(l-z)^2}$$

$$\frac{d^2y}{dz^2} + \frac{3lEI}{m\upsilon^2 z^2(l-z)^2} y = \frac{g}{\upsilon^2} \qquad (2)$$

(2) It is clear that the differential equation is not a simple equation with variable coefficients.

Such a variable coefficient is approximated by dividing differential equations into algebraic series In the first approximation, the ratio of the maximum dynamic deviation to the static deviation is defined as follows [5].

$$\mu = 1 + \frac{ml}{3EI} * \upsilon^2 \qquad (3)$$

In this last equation, we see a new factor - the effect of the speed of movement of the load. It can also be obtained in the quadrature solution of the differential equation (2). For that, let's write the equation (2) abbreviated.

$$\frac{d^2y}{dz^2} + k^2 y = h$$

Here $k^2 = 3lEI/m\upsilon^2(lz-z^2)^2$, $h = g/\upsilon^2$

(3) the characteristic equation of the left side of the differential equation

$$\lambda^2 + k^2 = 0, \quad \lambda = \pm ik$$

$$y_1 = C_1 \cos kz + C_2 \sin kz$$

$$y_2 = Ax, \quad \dot{y_2} = A, \quad \ddot{y_2} = 0$$

$$k^2 A = hA = \frac{h}{k^2}$$

If we put it instead

$$y = y_1 + y_2 = C_1 \cos[\frac{3lEI}{m\upsilon^2(lz-z^2)^2}]Z + C_2 \sin[\frac{3lEI}{m\upsilon^2(lz-z^2)^2}]Z + \frac{g}{\upsilon^2[3lEI/m\upsilon^2(lz-z^2)^2]}$$

(2) J. Boussinesq proposed the following transformation for the differential equation and proposed an equation with a constant coefficient [6]

$$\frac{1}{2}\ln\frac{z}{l-z} = \xi, \qquad y\frac{8\upsilon^2}{gl^2}\mathrm{ch}\xi = \eta$$

As a result, we get the following differential equation

$$\frac{d^2\eta}{d\xi^2} + k^2\eta = \frac{2}{\mathrm{ch}\xi} \qquad (4)$$

Here $k^2 = \left|\frac{12EI}{ml\upsilon^2} - 1\right|$

Other studies have been conducted with the problems posed in this context. But its practical meaning is not so important, because the mass of the device cannot be ignored compared to the mass of the load.

The first term on the right side of the equation expresses the effect of the load, and the second term expresses the effect of its inertial force. At the same time, let's give the formula of known bending moment in resistance of materials

$$M = \frac{EI}{\varrho} \qquad (5)$$

From equations (4) and (5), let's subtract the radius of curvature ϱ of the bent beam

$$\varrho = \frac{EI}{M}, M = \frac{pl}{4} + \frac{pl}{4g}\frac{\upsilon^2 M}{EI};$$

$$M - \frac{pl\upsilon^2}{4gEI}M = \frac{pl}{4}; \quad M\left(1 - \frac{pl\upsilon^2}{4gEI}\right) = \frac{pl}{4}.$$

$$M\left(\frac{4gEI - pl\upsilon^2}{4gEI}\right) = \frac{pl}{4}; \qquad M = \frac{pl}{4 - \frac{pl\upsilon^2}{EIg}}$$

Finally, we can observe the effect of the speed of the load on the bending moment and consider whether the critical speed can be found

$$4 - \frac{pl\upsilon_{kp}^2}{EIg} = 0 \qquad \upsilon_{kp} = \sqrt{\frac{4gEI}{pl}}$$

At this critical speed, we see that the bending moment and deflection of the beam tends to infinity.

In general, when considering the problem of moving loads acting on an elastic system, the mass of the load and the mass of the structure should be taken into account. If we do not take into account any of these two masses, then we get the solution of the problem in the form of an approximation.

But among these there are special cases, that is, the two factors mentioned above cannot be taken into account. Consider the following problem as an example. Let the load of weight p¯ move along the beam located on a homogeneous elastic base with speed υ. (2 – picture)

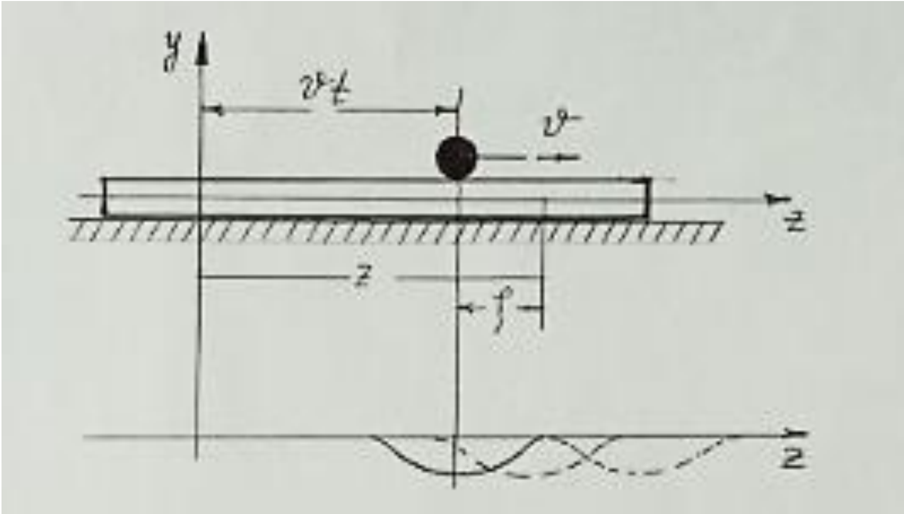


Figure 2. Effect of moving load

This problem differs from the problems discussed above, the bending of the beam remains constant during movement, and the load moves horizontally. (Fig. 2). The appearance of bending of the axis of the beam does not change, but depends on the speed of movement of the uniform load. Therefore, the bending of the beam is like a repeating wave-like bending.

Since the vertical coordinate of the load remains unchanged, its vertical acceleration is equal to zero, and the pressure of the load on the beam is equal to Pweight. That is why the report is known for such features.

Assuming linear deformation of the base of the beam, then the resistance reaction of the base of the beam is expressed by the following relationship.

$$z = -ky \tag{6}$$

Here y is the deviation, k is the proportionality coefficient describing the stiffness of the beam base.

Let's run the Z axis along the base of the beam, it starts from the origin of fixed coordinates, t is time, m is the mass per unit length of the beam, and the differential equation of bending of the beam is expressed as follows

$$EI\frac{\partial^4 y}{\partial z^4} = -m\frac{\partial^2 y}{\partial t^2} - ky \tag{7}$$

The right-hand side of equation (7) represents the intensity of the load and consists of two terms: the inertia of the load and the reaction of the elastic base.

Let's rewrite the differential equation (7) in the following form

$$\frac{\partial^4 y}{\partial z^4} + 2a\frac{\partial^2 y}{\partial t^2} + b^2 y = 0 \tag{8}$$

here

$$2a = \frac{m}{EI}, b^2 = \frac{k}{EI} \tag{9}$$

$$y = f(z - \upsilon t) \tag{10}$$

here $z - \upsilon t = \xi$

and it indicates the abscissa of the cross-section of the beam. The moving coordinate that faces the load is calculated from the beginning. Our goal is to explain or define the type of the newly introduced f function. If we denote the differentiation of this function by the argument ξ=z-υ in the form of a bar, then we express the derivatives of the required function y as follows

$$\frac{\partial^4 y}{\partial z^4} = f'^{V}(\xi), \qquad \frac{\partial^2 y}{\partial t^2} = \upsilon^2 f''(\xi) \tag{11}$$

Then we can write the independent differential equation of the form (8) in the form of the following ordinary differential equation

$$f'^{V} - 2a\upsilon^2 f'' + b^2 f = 0 \tag{12}$$

(12) is a fourth-order differential equation with a constant coefficient. Let's create a characteristic equation to solve it

$$\lambda^4 - 2a\upsilon^2\lambda^2 + b^2 = 0, \lambda^2 = h$$

$$h^2 - 2a\upsilon^2 h + b = 0$$

$$h_{1,2} = \frac{2a\upsilon^2 \pm \sqrt{4a^2\upsilon^4 - 4b^2}}{2} = \frac{2a\upsilon^2 \pm 2\sqrt{a^2\upsilon^4 - b^2}}{2}$$

$$h_1 = a\upsilon^2 + \sqrt{a^2\upsilon^4 - b^2}, h_2 = a\upsilon^2 - \sqrt{(a\upsilon^2 + b)(a\upsilon^2 - b)}$$

$\xi > 0$ taking into account, we write the solution of the function f classically

$$f = e^{-\alpha\xi}(C_1 \sin\beta\xi + C_2 \cos\beta\xi) + e^{\alpha\xi}(C_3\beta\xi + C_4 \cos\beta\xi) \tag{13}$$

Here, α and β are determined by the following formulas using the coefficients of equation (12).

$$\alpha = \sqrt{\frac{b - a\upsilon^2}{2}}, \beta = \sqrt{\frac{b + a\upsilon^2}{2}} \tag{14}$$

And for the second part of the wave, i.e. when ξ>0, the axis bending equation is defined in the form (14) above.

$$f_1 = e^{-\alpha\xi}(D_1 \sin\beta\xi + D_2 \cos\beta\xi) + e^{\alpha\xi}(D_3 \sin\beta\xi + D_4 \cos\beta\xi) \qquad (15)$$

To determine the eight unknown constants $D_1, D_2, D_3, D_4, C_1, C_2, C_3, C_4$ equations (13) and (15), we use initial conditions.

$\Xi = \pm\infty$ deviations of f and f_1 must be finite when tending, and when ξ=0

$$f(0) = f_1(0), \quad f'(0) = f_1'(0)$$

$$f''(0) = f_1''(0), \quad f_1'''(0) - f'''(0) = \frac{p}{EI} \qquad (16)$$

According to the first condition, i.e. when ξ=±∞

$$D_1 = D_2 = C_3 = C_4 = 0$$

And to determine the remaining constants, we use conditions (16), for which we need to determine the first, second, and third order derivatives of equations (13) and (15).

$$f_1' = -\alpha e^{-\alpha\xi}(D_1 \sin\beta\xi + D_2 \cos\beta\xi) + \alpha e^{\alpha\xi}(D_3 \sin\beta\xi + D_4 \cos\beta\xi) + e^{-\alpha\xi}(D_1\beta \cos\beta\xi - D_2\beta \sin\beta\xi) + e^{\alpha\xi}(D_3\beta \cos\beta\xi - D_4\beta \sin\beta\xi)\ ,$$

$$\begin{aligned} f_1'' = {} & +\alpha^2 e^{-\alpha\xi}(D_1 \sin\beta\xi \\ & + D_2 \cos\beta\xi) \\ & + \alpha^2 e^{\alpha\xi}(D_3 \sin\beta\xi + D_4 \cos\beta\xi) - \alpha e^{\alpha\xi}(D_1\beta \cos\beta\xi - D_2\beta \sin\beta\xi) + \alpha e^{\alpha\xi}(D_3\beta \cos\beta\xi \\ & - D_4\beta \sin\beta\xi) - \alpha e^{-\alpha\xi}(D_1\beta \cos\beta\xi - D_2\beta \sin\beta\xi) \\ & + \alpha e^{\alpha\xi}(D_3\beta \cos\beta\xi - D_4\beta \sin\beta\xi) \\ & + e^{-\alpha\xi}(-D_1\beta^2 \sin\beta\xi - D_2\beta^2 \cos\beta\xi) + e^{\alpha\xi}(-D_3\beta^2 \sin\beta\xi - D_4\beta^2 \cos\beta\xi). \end{aligned}$$

$$\begin{aligned} f_1''' = {} & -\alpha e^{-\alpha\xi}(D_1 \sin\beta\,\xi + D_2 \cos\beta\xi) + \alpha^2 e^{-\xi}(D_1\beta \cos\beta\xi - D_2\,\beta \sin\beta\xi) + \\ & +\alpha^3 e^{\alpha}\xi(D_3 \sin\beta\xi + D_4 \cos\beta\,\xi) + \alpha^2 e^{\alpha\xi}(D_3\beta \cos\beta\xi - D_4\beta \sin\beta\xi) - \alpha^2 e^{\alpha\xi}(D_1\beta \cos\beta\xi - D_2\,\beta \sin\beta\xi) - \\ & \alpha e^{2\xi}(-D_1\beta^2 \sin\beta\xi - D_2\beta^2\,\xi) + \alpha^2 e^{\alpha\xi}(D_3\beta \cos\beta\xi - D_4\beta \sin\beta\xi) + \alpha e^{\alpha\xi}(-D_3\beta^2 \sin\beta\xi - D_4\beta^2 \cos\beta\xi) - \\ & \alpha^2 e^{-\alpha\xi}(D_1\beta \cos\beta\xi - D_2\beta \sin\beta\xi) - \alpha e^{-\alpha\xi}(-D_1\beta^2 \sin\beta\xi - D_2\beta^2 \cos\beta\xi) + \alpha^2 e^{\alpha\xi}(D_3\beta \cos\beta\xi - D_4\beta \sin\beta\xi) + \\ & \alpha e^{\alpha\xi}(-D_3\beta^2 \sin\beta\xi - D_4\beta^2 \cos\beta\xi) + -\alpha e^{-\alpha\xi}(-D_1\beta^2 \sin\beta\xi - D_2\beta^2 \cos\beta\xi) + e^{\alpha\xi}(-D_1\beta \cos\beta\xi - \\ & D_2\beta \sin\beta\xi) + \alpha e^{\alpha\xi}(-D_3\beta^2 \sin\beta\xi - D_4\beta^2 \cos\beta\xi) + e^{\alpha\xi}(-D_3\beta \cos\beta\xi - D_4\beta \sin\beta\xi)\ . \end{aligned}$$

Similarly, find the first, second, and third derivatives of the function f C_1, C_2, D_3, D_4, we find constants

$$C_1 = -\frac{\rho}{2EJ\beta(d^2+\beta^2)}$$

$$D_4 = C_1 = -\frac{\rho}{2EJ\beta(d^2+\beta^2)}$$

$$D_3 = \frac{\rho}{2EJ\beta(d^2+\beta^2)}$$

Therefore, since the vibration of the beam is wave-like, it will be symmetrical as shown in Figure 2 above, and to analyze the obtained results, it is enough to consider the case φ>0, i.e.

$$f = \frac{\rho e^{-d\varphi}}{2EJ\beta(d^2+\beta^2)d*\beta}(d \sin\beta\varphi + \beta \cos\beta\varphi) \quad (17)$$

Here we are interested in the deviation under load, i.e

$$\varphi = 0$$

$$f(0) = \frac{\rho}{2EJd(d^2+\beta^2)} \qquad (18)$$

In order to find out the effect of speed υ on f(0), we put values of parameters α and β calculated by formula (14) into the last equation. There

$$f(0) = \frac{\rho}{EY\sqrt{\frac{b-a\vartheta^2}{2}*\left(\frac{b-a\vartheta^2}{2}\right)*\frac{b+a\vartheta^2}{2}}} =$$

$$= -\frac{\rho}{2EY\sqrt{\frac{b-a\vartheta^2}{2}*\left(\frac{b^2-a^2\vartheta^2}{4}\right)}} = -\frac{\rho}{EJbV2(b-a\vartheta^2)} \quad (19)$$

Statistical deviation due to the force exerted on a beam of infinite length placed on an elastic base when υ = 0, that is, when the load is stationary f_{cm} we can get. $f(0)/f_{cm}$ f we consider the relationship, we get the concept of dynamic coefficient. If we consider the formula (9), we get the following relation.

$$\mu = \frac{f(0)}{f} = \sqrt{\frac{1}{1-\frac{m\vartheta^2}{2\sqrt{kEJ}}}} = \sqrt{\frac{2\sqrt{kEJ}}{2\sqrt{kEJ-m\vartheta^2}}} \qquad (20)$$

As can be seen from the formula (20), we see that the dynamic coefficient μ increases as the speed υ increases.

$$\vartheta = \sqrt{\frac{2\sqrt{kEy}}{m}} \qquad (21)$$

When it is equal, we notice that this coefficient tends to infinity. The speed found by the formula (21) is called the test speed.

At speeds lower than this test speed, the above-mentioned chase waves are in the form of a distorted sine and their amplitudes decrease. parameter is zero.

Accordingly, the curve (17) begins to fade with time, and tends to a bald sinusoid at the test speed. It should be noted here that it is better not to be completely satisfied with the obtained results, because the inertial and resistance forces of the base under the beam are not considered in these equations and formulas. Nevertheless, the obtained solutions could qualitatively show an increase in the deviation as the speed of the load increases.

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