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CONCEPTS OF VISCOELASTIC MATERIALS AND CONSIDERATIONS OF NUCLEAR RELAXATION PARAMETERS

Abstract*: In this work, the problem of finding solutions to the general motion differential equations of a deformable rigid body, varying according to the harmonic law and satisfying certain boundary conditions, is considered.*

Key words*: viscoelastic materials, nuclear relaxation, viscoelastic analogy, Kelvin-Foyga model, dissipation energy, hereditary-elastic theory.*

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Introduction

Today, reducing the negative impact of microvibrations generated during wave propagation and increasing the strength of parts and devices is of great scientific and practical importance in the world. Due to the widespread use of modern techniques and technologies for structural materials with unique physical and mechanical properties, the creation of a methodology for studying the properties of wave propagation in plate and cylindrical viscoelastic shells in contact with the medium is a pressing issue.

Viscoelastic materials include polymers, composites, concrete, rocks, metals at high temperatures, and others. It is known from the literature that at ordinary temperatures (from -20 to +200°C), metals possess elastic properties [11,12]. If the temperature is above +20000C, it exhibits viscoelastic properties.

Plastic is close to elastic properties at 00°C. At temperatures above +500C, it exhibits viscoelastic properties. Therefore, instead of dividing materials into elastic and viscoelastic, it is possible to talk about the elastic and viscoelastic state of the material depending on temperature or other factors. Other properties of materials include factors arising under the influence of external loading.

For example, a material made of elastic materials can absorb external impact or dissipate energy in a mechanical system during vibration. The presence of new variable properties (time t) makes it more difficult to calculate structures made of viscoelastic materials compared to the calculations of the theory of elasticity. The main difficulty here is to establish the relationship between the stresses and deformations of a viscoelastic body [9,10].

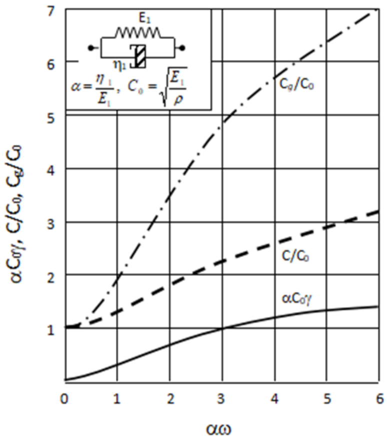

Figure 1. Kelvin-Foig's model. Dependence of the phase velocity c, the group velocity cg, and the attenuation coefficient γon the frequency ω/2π for sinusoidal waves

There are several approaches to establishing these relationships. One of them is based on simplified mechanical models (Foygt and Maxwell models), in which the elastic properties are described according to Hooke's law, and the viscous properties according to Newton's rheological law. At the same time, these models do not reflect the actual motion of the loaded material made of a viscoelastic material and do not satisfactoryly coincide with the experiment. For example, the Foigtt model does not have the property of stress relaxation (Fig. 1). and Maxwell's model only implies a linear law of change in deformation over time, which takes into account relaxation.

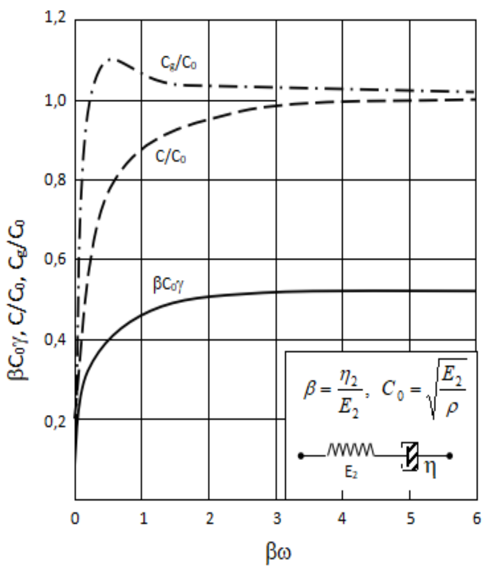

Figure 2. Maxwell model. Dependence of the phase velocity c, group velocity C_g, and attenuation coefficient γon the frequency $\omega/2\pi$ for sinusoidal waves

By generalizing the Foigth and Maxwell models, it is possible to obtain more complex combined models (generalized Kelvin-Foigth-Meier model), which in many cases turn out to be very suitable for describing the processes of slippage and loosening (Fig. 1 and Fig. 2). The use of the viscoelastic analogy allows replacing the modulus of displacement of the material and the modulus of elasticity with the operators G and E when deriving the basic equations, therefore [8]:

$$G = G'\left(1 + \eta_1 \frac{\partial}{\partial t}\right), E = E'(1 + \eta_2 \frac{\partial}{\partial t})$$

Standard viscoelastic body

$$E\left(\frac{\partial}{\partial t}\right) = \frac{E''_\partial + E''_\partial \frac{\partial}{\partial t}}{1 + \nu_2 \frac{\partial}{\partial t}},$$

$$G\left(\frac{\partial}{\partial t}\right) = \frac{G'_\infty + G''_\partial \nu_1 \frac{\partial}{\partial t}}{1 + \nu_1 \frac{\partial}{\partial t}}$$

Based on solutions involving the time factor, harmonic processes can be written $e^{i\Omega t}$ in harmonic form, and viscoelastic operators in a more general form

$$G(e^{i\Omega t}) = (G''_\gamma + iG''_i)e^{i\Omega t}, E(e^{i\Omega t}) = (E''_\gamma + iE''_i)e^{i\Omega t}$$

In this case, the real and imaginary parts of the complex module usually depend on the frequency values. A clear solution to the problem of wave propagation of multilayer structures is possible in some special cases

$$\bar{\lambda}_j = \frac{\nu_j \bar{E}_j(\omega)}{(1+\nu_j)(1-2\nu_j)}; \quad \bar{\mu}_j = \frac{\nu_j \bar{E}_j(\omega)}{2(1+\nu_j)}$$

$\bar{u}(u_x, u_a, u_r)$ - displacement of medium points

ν_j- Puasson koeffitsiyenti, $\bar{E}_j(\omega)$ -complex modulus of elasticity (Yung modulus), which has the form:

$$\bar{E}_j(\omega) = E_j(\omega)[1 + i\eta_j(\omega)]$$

$$gradf = \frac{\partial f}{\partial r}\bar{k}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\bar{k}_\theta + \frac{\partial f}{\partial x}\bar{k}_x;$$

$\bar{k}_r, \bar{k}_\theta, \bar{k}_z$- unit vectors;

$$div\bar{U} = \frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_x}{\partial x}, \lambda = \frac{2\nu G}{1-2\nu}, \quad \mu = G$$

In many cases, viscosity and energy loss are introduced by relating the modulus of elasticity to frequency in a complex form:

$$E(\omega) = E_{min} + E_{max}\left[1 - \frac{1}{1+(\beta\omega)^n}\right]$$

$$\eta\omega = \frac{n\pi E_{min}(\beta\omega)^n}{2E(\omega)[1+(\beta\omega)^n]^2}$$

For ordinary materials, the modulus of elasticity is taken as follows.

$$E'' = E(\omega)\eta(\omega) = \frac{n\pi E_{max}(\beta\omega)^n}{2[1+(\beta\omega)^n]^2}$$

In general, there are different approaches to the selection of the relaxation kernel of the hereditary-elasticity theory. The relationship between stresses and displacements satisfies Hooke's law with operator coefficients [6, 7].

$$\sigma_{ijk} = \tilde{\lambda}_k \theta_k \delta_{ij} + 2\tilde{\mu}_k \varepsilon_{ijk}$$

Here λ_k, μ_k - operator modulus of elasticity, ε_{ijk} - elements of the strain tensor,θ- volumetric strain,

$$\tilde{\lambda}_k f(t) = \lambda_{0k}\left[f(t) - \int_0^t R_{\lambda k}(t-\tau)f(\tau)d\tau\right];$$

$$\tilde{\mu}_k f(t) = \mu_{0k}\left[f(t) - \int_0^t R_{\mu k}(t-\tau)f(\tau)d\tau\right];$$

$f(t)$- arbitrary function of time;

$R_{\lambda k}(t-\tau), R_{\mu k}(t-\tau)$- relaxation nucleus;

λ_{ok}, μ_{0k} --instantaneous modulus of elasticity

The kernel of the integral operator of the fractional-exponential function of Y.N. Rabotnov is expressed as follows

$$A\text{Э}_\alpha^{(n)}(-\beta, t) = At^\alpha \sum_{n=}^{\infty} \frac{(-\beta)^n t^{n(1+\alpha)}}{\Gamma[(n+1)(1+\alpha)]} \qquad (2)$$

$$\tilde{\nu}[f(t)] = \nu_0 + \frac{1-2\nu_0}{2}\Gamma_k^*$$

$$\Gamma^* f(t) = A\int_{-\infty}^{t} \text{Э}_\alpha(-\beta, t-\tau)f(\tau)d\tau,$$

$$\beta > 0, -1 < \alpha \le 0, A < \beta$$

Here

$G(j) = \int_0^\infty y^{j-1}\exp(-y)dy$ - gamma function, E_0, ν_0–instantaneous values of Young's modulus and Poisson's ratio, A,β,α - viscosity parameters of the material, $f(t)$ - arbitrary function of time.

The following given values are used to obtain numerical results:

$$\alpha = 0{,}100237, A = 0{,}07316c^{-1},$$
$$\beta = 0{,}05024c^{-1}, \delta_{max} = 57 * 10^{-5},$$
$$c = 9{,}9816MPa$$

Currently, the most effective way to take into account the viscosity property is the three-parameter Rzhanitsyn-Koltunov relaxation kernel, which takes the Poisson's ratio as constant:

$$R_i(t) = A_i e^{-b_i t}/t^{1-a_i}$$

Here A_j- amplitude of the viscosity kernel, $\alpha_j$and β_j- parameters of the viscosity kernel [5]. In calculations

$$A = 0{,}048; \ \beta = 0{,}05; \ \alpha = 0{,}10$$

The usual conditions $R_j(t-\tau)$ for integrability, continuity ($t = \tau$ except for a point), definite sign, and monotonicity of the action function are as follows:

$$R \succ 0, \frac{dR(t)}{dt} \le 0, \ 0 \prec \int_0^\infty R(t)dt \prec 1$$

Such problems are one of the main problems of the mechanics of deformable solids. In the eighties of the last century, the scientific school of M.A. Koltunov and I.E. Troyanovskiy laid the foundation for the theory of quasistatic deformation of dissipatively- homogeneous and inhomogeneous mechanical systems. This theory was later generalized for solving dynamic problems in the works of V.P. Mayboroda, M.M. Mirsaidov, I.I. Safarov, V.M. YAganov, A. Balakirov, and others.

Systems with identical viscoelastic properties of elements are called dissipatively homogeneous, and systems with different rheological characteristics are called dissipatively inhomogeneous.

Systems whose elements have the same viscoelastic properties are called dissipatively homogeneous, and systems with different rheological characteristics are called dissipatively inhomogeneous.

Issues of wave propagation in elastic waveguides in the form of rods, plates, and tubes have been considered in many literature sources [3, 4]. In terms of their characteristics, these waves are similar to Lamb waves and the obtained SH-polarization waves. The main goal of this chapter is to study the dissipative properties of long cylindrical mechanical systems during the propagation of harmonic waves. In most cases, the propagation of an elastic wave in bounded bodies represents the phenomenon of dispersion, in which the speed of propagation of infinite monochromatic sinusoidal waves is a function of the wavelength (or frequency). Dispersion plays an important role in the study of pulse voltage waves.

The momentum can be considered as sinusoidal (with different frequencies) components of the Fourier integral (for bodies moving at different speeds).

From this it follows that the shape of the pulse changes as it begins to move from the point of origin, or, in other words, dispersion changes. When analyzing the phenomenon of dispersion, it is first necessary to determine the change in the wave propagation velocity (phase velocity C) relative to the wavelength λ [1,2].

This part of the analysis consists in finding solutions to the general differential equations of motion of a deformable solid body, which change according to a harmonic law and satisfy certain boundary conditions. In practice, equations expressing boundary conditions are so complex that this stage can be completed only in some special cases when using exact equations. The velocity S is called the phase velocity, since it represents the distance traveled per unit time by a point having a constant phase (say, a maximum or minimum). It necessarily coincides with the velocity of propagation of the impulse, denoted by V, which is called the group velocity. To determine the group velocity V, a lying curve of the impulse is drawn, and the distance traveled by this lying curve per unit time is measured. It can be shown that the grou p velocity V:

$$V = C - \lambda\frac{dc}{d\lambda} = C + \omega\frac{dc}{d\omega} \qquad (3)$$

is equal to. $Here\ S, \lambda, \omega, dc/d\lambda$ and $dc/d\omega$ are the average values over the frequency interval representing the main part of the pulse. Since wave propagation is for directions, one direction is chosen. For example, k_1 the wave number, then $\omega = k_1$S - complex frequency, $S\,(S = S_R + iC_I)$- complex phase velocity. To determine their physical meaning, let's consider two cases:

1) $k = \gamma_R, S = S_R + iC_I$ - then the amplitude of the solution (3) takes the form of a sinusoid with respect to the x-coordinate, attenuating with respect to time;

2) $k = \gamma_R + i\gamma_I; S = S_R$, - then the oscillation will be stable at all points x, but will decay along the x-axis.

3) In both cases, γ_R either the imaginary parts of C_I characterize the intensity of the dissipative process.

If the velocity C increases with a frequency C<V, then the lying curve will be greater than the velocity of the individual frequencies that make it up. If the velocity C decreases with a frequency C>V, then the reverse situation is observed. Dispersion is an important factor for several reasons, but it is important that the pulse energy propagates at a speed of V. Also, the dispersion of bulk waves follows from many theories proposed to account for wave attenuation, but in practice it is not observed.

Summary

In conclusion, we analyzed the literature devoted to finding the dispersion equation expressing the propagation of waves in a cylindrical shell with a liquid in the general case using the Lame equation. The concepts of wave propagation and wave dispersion in an elastic medium and the literature devoted to their study were analyzed. Waves in the semi-elastic plane and the contact of two media (Stone) were also analyzed. An analysis of the literature devoted to obtaining a dispersion equation describing wave propagation, taking into account the viscoelastic properties of materials, is also presented.

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