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FREE VIBRATIONS OF A MECHANICAL SYSTEM FORMED FROM MASSES SUSPENDED ON CIRCULAR THIN ALLOY-ELASTIC PLATES

Abstract: *The free vibrations of a mechanical system consisting of masses suspended from circular thin elastic plates are considered. In recent years, glass-fiberglass has been widely used as a material for the housings of radio electronic devices (REA). Therefore, these materials were used in theoretical calculations. Taking into account the rheological properties of the plate allows reducing the real and imaginary parts of the frequency by 10%. Thus, a mathematical formulation and solution methodology for solving the problem of free vibrations of dissipative homogeneous and inhomogeneous mechanical systems of a circular plate with attached masses are developed. For elements of a dissipative mechanical system, the relationship between stresses and deformations satisfies the linear hereditary Bolsmann-Voltera relations. The variation of frequencies and damping coefficients depending on the dimensionless frequency coefficient is given.*

Key words: *circular thin viscous-elastic plate, suspended mass, free vibrations, frequency, viscosity.*
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Introduction

Factors affecting the structural elements of moving bodies include vibration and shock loads, acoustic effects, etc. Aerodynamic forces increase the vibrational vibrations of elements and parts of moving devices. When designing such structures, dynamic calculations are performed to determine the safety limits of the structure, calculate the resonant frequencies and factors affecting the details of the structure. The calculation of radio technical devices is complicated, since such structures consist of elements of complex shape with a large number of small parts. Methods for calculating linear elastic free and forced vibrations of a mechanical system are close to complete using analytical and numerical methods. A lot of literature is devoted to the practical application of vibration, and numerical calculations are also widely used. In particular, the theory of calculating elastic vibrating systems from vibrations is covered in detail in the literature. The problem of calculating free frequencies and vibration forms in a linear elastic mechanical system with finite degrees of freedom has been fully solved using standard programs. The problems of calculating the dynamic state of elastic mechanical systems with distributed parameters are mainly solved using numerical methods.

2. Methods

2.1. Problem formulation and solution methods.

Structures with masses attached to thin deformable plates or shells are widely used in aircraft, unmanned aerial vehicles, manufacturing plants, and construction. The problem is to find the free vibrations of a plate with masses suspended. Here, examining the frequencies in the systems allows for the prediction of resonance phenomena.

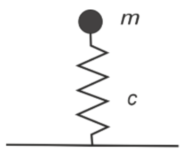


Figure 1. Mass mounted on a plate

Let's assume radius R, thickness h, density of the material ρ and the operator is a cylindrical unit in the form $\tilde{D}$ to a dense elastic plate with $m_k (k=1,...,N)$ Let it hang. The operator's virginity is as follows

$$\tilde{D} f(t)=D_0\left[f(t)-\int_0^t R_D(t-\tau)f(\tau)d\tau\right]; \qquad (1)$$

$$\tilde{c}_\kappa f(t)=c_{0\kappa}\left[f(t)-\int_0^t R_{c\kappa}(t-\tau)f(\tau)d\tau\right]$$

here $f(t)$– arbitrary function of time, $R_D(t-\tau)$ and $R_{c\kappa}(t-\tau)$– relaxation nuclei, $D_0=E_0h^3/(12(1-\nu^2))$, $c_{0\kappa}$– is the instantaneous elastic modulus. Then the differential equations of motion of the plate and the mass suspended from it are as follows:

$$\tilde{D}\nabla^4 w+\rho h\ddot{w}=\sum_{k=1}^{N}\tilde{c}_k(y_k-w_k)\delta_k+q(r,\theta,t), \qquad (2)$$

$$m_k\ddot{y}_k+\tilde{c}_k(y_k-w_k)=p_k(t),$$

$$\bar{D}f(t)=D_0\left[1-\Gamma_D^C(\omega_R)-i\Gamma_D^S(\omega_R)\right]f(t)=D_0\Gamma_D,$$

$$\bar{c}_\kappa f(t)=c_{0\kappa}\left[1-\Gamma_{c\kappa}^C(\omega_R)-i\Gamma_{c\kappa}^S(\omega_R)\right]f(t)=c_{0\kappa}\Gamma_{c\kappa}, \qquad (3)$$

here

$$\Gamma_D^C(\omega_R)=\int_0^\infty R_D(\tau)\cos\omega_R\tau\,d\tau,\Gamma_{ck}^C(\omega_R)=\int_0^\infty R_{c\kappa}(\tau)\cos\omega_R\tau\,d\tau$$

$$\Gamma_D^s(\omega_R)=\int_0^\infty R_D(\tau)\sin\omega_R\tau\,d\tau,\quad \Gamma_{ck}^s(\omega_R)=\int_0^\infty R_{c\kappa}(\tau)\sin\omega_R\tau\,d\tau$$

Then the differential equation (2) takes the following form

$$\bar{D}\nabla^4 w+\rho h\ddot{w}=\sum_{k=1}^{N}\bar{c}_k(y_k-w_k)\delta_k+q(r,\theta,t),$$

$$m_k\ddot{y}_k+\bar{c}_k(y_k-w_k)=p_k(t). \qquad (4)$$

When solving a problem, boundary and initial conditions are imposed.

$$L_1(W)\big|_{r=R}=0, L_2(W)\big|_{r=R}=0, \qquad (5)$$

$$w\big|_{t=0}=0,\frac{\partial w}{\partial t}\big|_{t=0}=0, \qquad (6)$$

$$y_k\big|_{t=0}=0,\frac{\partial y_k}{\partial t}\big|_{t=0}=0$$

We use condition (5) to find arbitrary constants. If the mass m_k hanging points (r_k,θ_k) if we write down the displacements of , it will be as follows:

$$W_j(r_j,\theta_j)=\sum_{n=0}^{\infty}\left[A_nJ_n(\beta_b r_j)+B_nI_n(\beta_b r_j)\right]\cos(n\theta)- \qquad (7)$$

$$-\sum_{k=1}^{N}\sum_{n=0}^{\infty}\sum_{j=1}^{\infty}\frac{\bar{c}_k(Y_k-W_k)V_{njk}}{D_0s_{nj}(\beta_b^2-\alpha_{1nj}^4)}V_{jik}(r,\theta), i=1,2,..N.$$

If we subject the obtained solutions to boundary conditions $\bar{c}_k(Y_k-W_k)$ if we take it out of the equation and simplify it $V_{nj}(r,\theta)$ We obtain the following system of algebraic equations with respect to the variable.

$$A_nL_1\left[J_n(\beta_b r)\right]\big|_{r=R}+B_nL_1\left[I_n(\beta_b r)\right]\big|_{r=R}=0, \qquad (8)$$

$$A_nL_2\left[J_n(\beta_b r)\right]\big|_{r=R}+B_nL_2\left[I_n(\beta_b r)\right]\big|_{r=R}=0, n=1,2..N.$$

In this case, from (8) $A_{ns}=B_{ns}=0$ this system (N-1)- from the equation $Y_{js}=C_{js}Y_{Ns}$ (j=1,2…N; C_{Ns}=1).

Thus, based on the obtained solutions, we find the form of oscillations ($W_s(r,\theta)$ $W_s(r,\theta)$)

$$W_r(r,\theta)=-\frac{\beta_s^4}{\rho h}\sum_{k=1}^{N}C_{ks}m_k\sum_{n=0}^{\infty}\sum_{i=1}^{\infty}\frac{V_{nik}V_{nij}}{s_{ni}(\beta_s^4-\alpha_{1nj}^4)},V_{ks}=C_{ks}(C_{Ns}=1). \qquad (9)$$

The equation of the characteristic amplitudes Y_{km} and Y_{kj} accordingly Y_{kj}, Y_{km} Multiplying by , subtracting from the second equation, and $Y_{mk}W_{kj}-Y_{kj}W_{km}$ If we remove from the equation, then $\omega_m\neq\omega_j$ Since we derive the orthogonality condition

$$\int_0^R\int_0^{2\pi} W_m W_j r dr d\theta + \sum_{k=1}^{N}\frac{m_k}{\rho h}Y_{km}Y_{kj} = 0, m \neq j \quad (10)$$

The general solution of equation (2) is as follows:

$$w(r,\theta,t) = \sum_{m=1}^{\infty} q_m(t)W_m(r,\theta), y_k(t) = \sum_{m=1}^{\infty} q_m(t)Y_{km}, k = 1,...,N \quad (11)$$

If we substitute the solution (11) into system (2), then we obtain the following system

$$\sum_{m=1}^{\infty}\left(\ddot{q}_m + \omega^2 q_m\right)\rho h W_m = q(r,\theta,t),$$
$$\sum_{m=1}^{\infty}\left(\ddot{q}_m + \omega^2 q_m\right)m_k Y_m = P_k(t), \ k = 1,2,...N \quad (12)$$

The first equation of this system is W_k If we multiply by and integrate over the surface of the plate, and then sum over k in the second equation and use the orthogonality condition, we obtain the following equations:

$$\ddot{q}_j + \omega_j^2 q_j = Q_j(t); \ j = 1,2... \quad (13)$$

here $Q_j(t)$ - is a generalized force, which is determined by the following formula

$$Q_j(t) = \frac{1}{\rho h M_j}\left(\int_0^R\int_0^{2\pi} q(r,\theta,t)W_j(r,\theta) r dr d\theta + \sum_{k=1}^{n} P_k(t)Y_{kj}\right),$$

The general solution of equation (12) is as follows:

$$q_j(t) = A_j \cos\omega_j t + B_j \sin\omega_j t + \frac{1}{\omega_j}\int_0^t Q_j(\tau)\sin\omega_j(t-\tau)d\tau \quad (14)$$

here A_j and B_j is found from the initial conditions:

$$w(r,\theta,0) = w_0(r,\theta),$$
$$\dot{w}(r,\theta,0) = \dot{w}_0(r,\theta),$$
$$y_k\big|_{t=0} = y_{k0},$$
$$\dot{y}_k\big|_{t=0} = \dot{y}_{k0}, k = 1,2...N, \quad (15)$$

and the orthogonality condition takes the following form

$$A_j = \frac{1}{M_j}\left(\int_0^R\int_0^{2\pi} w_0 W_j r dr d\theta + \sum_{k=1}^{N}\frac{m_k}{\rho h}y_{k0}(t)Y_{kj}\right),$$
$$B_j = \frac{1}{\omega_j M_j}\left(\int_0^R\int_0^{2\pi} w_0 W_j r dr d\theta + \sum_{k=1}^{N}\frac{m_k}{\rho h}\dot{y}_{k0}(t)Y_{kj}\right), \quad (16)$$

To describe the viscous properties of system elements or deformation processes occurring at point connections, we use the Bolsmann-Voltaire linear inheritance theory:

$$\sigma_{mk}^n(t) = E_n\left[\varepsilon_{mk}^n(t) - \int_{-\infty}^{t} R^n(t-\tau)\cdot\varepsilon_{mk}^n(\tau)\,d\tau\right], \quad (17)$$

For the voltages to be a periodic function of time, the lower bound of the hereditary integral (17) is taken to be equal to minus infinity. If the lower bound is zero, the voltage gives aperiodic oscillations that decay with time.

Relaxation core $R(t-\tau)$ integrability, continuity ($t=\tau$ from outside), must have properties such as positive definiteness and monotonicity:

3. Numerical results and their analysis

Based on the analytical solution obtained above, Gauss's theorem, Laplace's theorem, the calculus of special functions, and Muller's methods were created. (C^{++}) Numerical results were obtained with the program. Mass of cargo M_1=0.1M_0, here M_0 The mass of the plate. The mass suspended from the plate is assumed to be point. The natural vibrations of the plate are studied. In this case, the natural frequencies are found. The physical and mechanical characteristics of the plate are as follows $E = 2.1\cdot10^{11}\Pi a; \rho = 7.85\cdot10^3 кг/m^3, \nu = 0.3$, radius R and thickness 0.5x0.001м, the parameters of the relaxation kernel are as follows: $A = 0{,}048; \ \beta = 0{,}05; \ \alpha = 0{,}1$.

Table 2.1. Comparison of plate bending frequency with work [11]

Plate dimensions h/R	numerical results obtained	[11] numerical results obtained in the work, Гц.	Error, %
0,20	1071,17	1079,00	0,14
0,40	1106,32	1197,18	3,12
0.60	1273,24	1265,78	3,35
1.0	1301,43	1307,86	4,07
2.0	1567,32	1563,1	4,65

It is clear that the results obtained at low frequencies 6% It gives an error of.

In recent years, fiberglass has been widely used as a material for housing radio electronic devices. Therefore, this material was used in theoretical calculations. The physical and mechanical properties of this material are as follows: – Young's modulus (E) -10^5 кгс/м^2, – density (ρ), - 1400 кг/м^3, – frequency (Ω), - 200. Let us assume that the amplitude of the external vibration load is 1 mm. If a mechanical system vibrates under the influence of vibration displacement, then $u_0(t) = A_0 \sin(\Omega t + \varphi)$ is taken in the form. Here A_0 - amplitude of rotational loading, Ω - periodic frequency of external vibration loading, φ - initial phase.a) tightness, b) The results are presented in Figure 2.2.

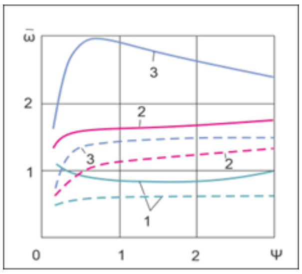
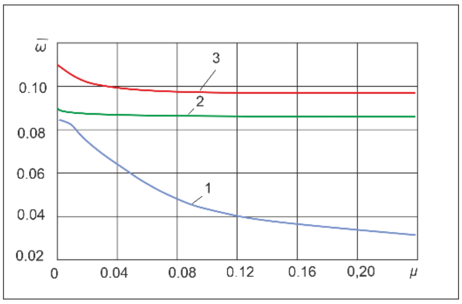


Figure 2. The variation of frequency with plate thickness is shown.

This figure 2.2 shows the variation of the fundamental frequency with the thickness of the plate. The variation of the frequency of the first and second modes of oscillations with the stiffness of the spring is shown. The mechanical effect obtained in the previous problem for a dissipative inhomogeneous mechanical system is found to be valid here as well. Increasing the number of accumulated masses increases the real and abstract parts of the frequency of free oscillations. 20% it leads to a reduction.

Increasing the number of suspended masses leads to a decrease in frequencies. The change in the abstract part of the frequencies is also close to the trend of changing the real frequency. In the next 3rd figure, the ratio of the suspended mass to the mass of the plate for fiberglass material when 2 symmetrical masses are suspended (for a hinged plate). The figure shows the change in the three frequency modes depending on the ratio of the masses.



3 – Fig. Variation of the real part of the frequency of the three modes depending on the mass ratio

2- Table 1. Variation of the natural frequency of the five modes with k

к	ω_1	ω_2	ω_3	ω_4	ω_5
1	0.08237	0.23371	0.42371	1.90235	2.52379
2	0.07143	0.23113	0.42143	0.92143	1.92353
3	0.06152	0.21628	0.43201	0.87152	1.91452
4	0.05474	0.12316	0.41414	0.76014	0.96147
5	0.04139	0.11432	0.41101	0.71392	0.84132
6	0.03127	0.11326	0.33801	0.71276	0.82278
7	0.02128	0.11248	0.31501	0.71128	0.81027

The variation of the natural frequency of the five modes with k is given in Table 2. Taking into account the rheological properties of the plate, the real and abstract parts of the frequency are 10% Thus, a mathematical formulation and solution methodology for solving the problem of free vibrations of dissipative homogeneous and inhomogeneous mechanical systems of a circular plate with attached masses have been developed. For elements of a dissipative mechanical system, the relationship between stresses and deformations satisfies the linear hereditary Bolsmann-Voltera relations.

Conclusions

1. A mathematical formulation, solution methodology, and algorithm for the problem of free and forced vibrations of dissipative uniform and inhomogeneous mechanical systems consisting of plates with point supports and attached masses have been developed. The reliability of the developed algorithm has been proven on the basis of numerical experiments;

2. The problems of (free and forced) vibrations of dissipative homogeneous (and non-homogeneous) shell systems with supports and accumulated mass are solved. To describe the dissipative properties of the entire system, the concept of global damping coefficient (GDC) introduced by I.E. Troyanovsky and I.I. Safarov was used.

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