Impact Factor:

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JIF	= 1.500	SJIF (Morocco)	= 7.184	OAJI (USA)	= 0.350

SOI: 1.1/TAS DOI: 10.15863/TAS

International Scientific Journal

Theoretical & Applied Science

p-ISSN: 2308-4944 (print) **e-ISSN:** 2409-0085 (online)

Year: 2025 **Issue:** 04 **Volume:** 144

Received: 29.04.2025
Accepted/Published: 30.04.2025

https://T-Science.org

Issue Article

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SMALL PARAMETER METHOD IMPLEMENTATION IN MAPLE ENVIRONMENT

Abstract: *The current development of computational systems is creating the conditions for the implementation of effective methods, distinguished by the speed of calculation and the minimization of the workload involved in the calculation. The article describes the development of a code for solving nonlinear differential equations by the small-parameter method in Maple, taking into account the advantages of computer-mathematical systems.*

Key words: *Cauchy problem, power series, small parameter, asymptotic expansion, Maple system.*

Language: *English*

Citation: Krakhmaleva, Yu.R., & Tursynova, T. (2025). Small parameter method implementation in maple environment. *ISJ Theoretical & Applied Science, 04 (144),* 107-114.

Soi: https://s-o-i.org/1.1/TAS-04-144-23 ***Doi***: crossref https://dx.doi.org/10.15863/TAS.2025.04.144.23

Scopus ASCC: *2604.*

Introduction

Perturbation theory[1], [2] is a tool for the study of dynamical systems. The method of small parameters is one of the methods of this theory. It is used to solve nonlinear differential equations[3] and was proposed by A. Poincaré. Decomposing the desired solution into a series by a small parameter is the essence of the method. This tends to involve calculating the first or second approximation[4],[5].

For the solution of the Cauchy problem, let us consider the method of small parameters:

$$\begin{cases} \dfrac{dy}{dx} = f(y,x,\varepsilon) \\ y(0,\varepsilon) = y_0 \end{cases}, \qquad (1)$$

where ε is a parameter that has variations in some neighborhood $\varepsilon = 0$. At $\varepsilon = 0$ the solution of the differential equation from (1) is known. It is required to find the solution at small values of $\varepsilon \neq 0$.

As it is known, the following theorem is used to find the solution of problem (1): If the functions $f(y,x,\varepsilon)$ and $\dfrac{\partial f(y,x,\varepsilon)}{\partial y}$ are continuous on all variables y, x, ε in D where $D = \{t \le a, |y - y_0| \le b, |\varepsilon| \le c\}$, then the solution of problem (1) is continuous on x and parameter at ε, where $T = \min\left(a, \dfrac{b}{M}\right), M : |f(y,x,\varepsilon)| \le M$ and $|\varepsilon| \le c$. [6]

When the problem $\varepsilon = 0$ is written in the form:

$$\begin{cases} \dfrac{dy_0(x)}{dx} = f(y_0, x, 0) \\ y_0(0) = y_0 \end{cases} \qquad (2)$$

It follows from the above theorem that at $t \in [0,T]$:

$$y(x,\varepsilon) = y_0(x) + \varepsilon(x,\varepsilon), \qquad (3)$$

where $\varepsilon(x,\varepsilon)\to 0$ at $\varepsilon\to 0$. Formula (3) is an asymptotic representation of the solution $y(x,\varepsilon)$ by a small parameter ε .[6]

Assuming a series expansion by powers of ε , the functions $f(y,x,\varepsilon)$:

$$f(y,x,\varepsilon)=f_0(y,x,0)+\varepsilon\cdot f_1(y,x,0)+\varepsilon^2\cdot f_2(y,x,0)+\varepsilon^3\cdot f_3(y,x,0)+\dots, \qquad (4)$$

where $f_k(y,x,0)$- Taylor coefficients at $k=0,1,2\dots$, the solution of problem (1) has a representation in the form of a power series:

$$y(x,\varepsilon)=y_0(x)+\varepsilon\cdot y_1(x)+\varepsilon^2\cdot y_2(x)+\varepsilon^3\cdot y_3(x)+\dots+\varepsilon^m\cdot y_m(x)+O(\varepsilon^m), \qquad (5)$$

where $y_0(x)$- is the solution of(1) at $\varepsilon=0$ (or tasks (2)) [6],[7],[8].

To find the coefficients $y_i(x)$, where $i=1,2\dots$, we need to substitute the expansion (5) into the equation of the system (1) and the initial condition. Then decompose the right-hand sides by powers of ε up to and including ε^m. Then the expressions at the same powers of $y_i(x)$ are made up and equated. Finally, we obtain a system of differential equations with initial conditions to determine $y_i(x)$. Sequentially, starting from $y_1(x)$, we find the solution of each system of differential equations with initial conditions $y_1(x)$, $y_2(x),\dots,y_m(x)$.[9],[10]

Let us apply the method of small parameter to solve the Cauchy problem:

$$\begin{cases}\dfrac{dy}{dx}=4\cdot\varepsilon\cdot x-y^2\\ y(1)=1\end{cases}. \qquad (6)$$

The functions of the right-hand side fulfill the conditions of the theorem , which means that it is possible to find a solution to problem (6) in the form (5), limiting the decomposition of the solution to the 3rd term:

$$y(x,\varepsilon)=y_0(x)+\varepsilon\cdot y_1(x)+\varepsilon^2\cdot y_2(x). \qquad (7)$$

Let's find

$$y'(x,\varepsilon):y'(x,\varepsilon)=(y_0(x))'+\varepsilon\cdot(y_1(x))'+\varepsilon^2\cdot(y_2(x))'. \qquad (8)$$

Let's substitute expression (6) for $y'(x,\varepsilon)$ into the left part of the equation of the system (6), expression (7) for $y(x,\varepsilon)$ into the right part of the equation of the system

$$(y_0(x))'+\varepsilon\cdot(y_1(x))'+\varepsilon^2\cdot(y_2(x))'=4\cdot\varepsilon\cdot x-(y_0(x)+\varepsilon\cdot y_1(x)+\varepsilon^2\cdot y_2(x))^2$$

or

$$(y_0(x))'+\varepsilon\cdot(y_1(x))'+\varepsilon^2\cdot(y_2(x))'=4\cdot\varepsilon\cdot x-((y_0(x))^2+\varepsilon\cdot y_0(x)\cdot y_1(x)+\varepsilon^2\cdot y_0(x)\cdot y_2(x)+$$
$$+\varepsilon\cdot y_1(x)\cdot y_0(x)+\varepsilon^2\cdot(y_1(x))^2+\varepsilon^3\cdot y_1(x)\cdot y_2(x)+\varepsilon^2\cdot y_2(x)\cdot y_0(x)+\varepsilon^3\cdot y_2(x)\cdot y_1(x)+\varepsilon^4\cdot(y_2(x))^2)$$

Let's transform the last expression:

$$(y_0(x))'+\varepsilon\cdot(y_1(x))'+\varepsilon^2\cdot(y_2(x))'=4\cdot\varepsilon\cdot x-(y_0(x))^2-2\cdot\varepsilon\cdot y_0(x)\cdot y_1(x)-\varepsilon^2\cdot(y_1(x))^2-$$
$$-2\cdot\varepsilon^2\cdot y_0(x)\cdot y_2(x)-\varepsilon^2\cdot(y_1(x))^2-2\cdot\varepsilon^3\cdot y_1(x)\cdot y_2(x)-\varepsilon^4\cdot(y_2(x))^2.$$

Write out the expressions at $\varepsilon^0,\varepsilon^1,\varepsilon^2$ left and right and equate them.

We have at ε^0: $(y_0(x))'=-(y_0(x))^2$,

at ε^1: $(y_1(x))'=4\cdot x-2\cdot y_0(x)\cdot y_1(x)$,

at ε^2: $(y_2(x))'=-2\cdot y_0(x)\cdot y_2(x)-(y_1(x))^2$.

The same is true for the initial conditioned expression, where the series is expanded around a small parameter:

$$y_0(1)+\varepsilon\cdot y_1(1)+\varepsilon^2\cdot y_2(1)=1.$$

The expansion coefficients define initial conditions for the equations that correspond to the same order degrees of the parameter. Since 123 is not present in the initial condition, $y_0(1)$ will be $y(1)$.

We have at ε^0: $y_0(1)=1$,

at ε^1: $y_1(1)=0$,

at ε^2: $y_2(1)=0$.

Thus, we obtained the Cauchy problem for finding $y_0(x), y_1(x), y_2(x)$:

$$\begin{cases}\dfrac{dy_0(x)}{dx}=(y_0(x))^2\\ y_0(1)=1\end{cases}; \qquad (9)$$

$$\begin{cases}\dfrac{dy_1(x)}{dx}=4\cdot x-2\cdot y_0(x)\cdot y_1(x)\\ y_1(1)=0\end{cases}; \qquad (10)$$

$$\begin{cases}\dfrac{dy_2(x)}{dx}=-2y_0(x)\cdot y_2(x)-(y_1(x))^2\\ y_2(1)=0\end{cases}. \qquad (11)$$

Solving the Cauchy problem (9):

$$y_0(x)=\frac{1}{x},$$

substitute in (10):

$$\begin{cases}\dfrac{dy_1(x)}{dx}=4\cdot x-\dfrac{2\cdot y_1(x)}{x}\\ y_1(1)=0\end{cases}$$

and find $y_1(x)$:

$$y_1(x)=x^2-\frac{1}{x^2}.$$

Functions $y_0(x)=\dfrac{1}{x}$, $y_1(x)=x^2-\dfrac{1}{x^2}$ substituting (11), we have the system:

$$\begin{cases}\dfrac{dy_2(x)}{dx}=-\dfrac{2}{x}y_2(x)-\left(x^2-\dfrac{1}{x^2}\right)^2\\ y_2(1)=0\end{cases},$$

solution of which:

$$y_2(x)=-\frac{x^5}{7}+\frac{2x}{3}+\frac{1}{x^3}-\frac{32}{21x^2}.$$

Substituting the found solutions $y_0(x)$, $y_1(x)$, $y_2(x)$ в $y(x,\varepsilon)$ we obtain the solution of the problem:

$$y(x,\varepsilon)=\frac{1}{x}+\varepsilon\cdot\left(x^2-\frac{1}{x^2}\right)+\varepsilon^2\cdot\left(-\frac{x^5}{7}+\frac{2x}{3}+\frac{1}{x^3}-\frac{32}{21x^2}\right).$$

The solution involves certain computational difficulties, as we can see. Let's set ourselves the task of implementing the method of small parameters in the system of computer mathematics. Its peculiarity is the minimisation of the labour intensity of the computational process with the accuracy of the calculations.

We will connect a specialised package and introduce the initial problem for the solution of differential equations:

$$restart;\ with(DETools);\ eq1 := diff(y(x),x) = 4\cdot\varepsilon\cdot x - y(x)^2;\ nus := y(1) = 1;$$

$$eq1 := \frac{d}{dx}\,y(x) = 4\,\varepsilon\,x - y(x)^2$$

$$nus := y(1) = 1$$

We write the solution of the problem in the form of a power series on the parameter ε:

$$y(x) := y0(x) + \varepsilon\cdot y1(x) + \varepsilon^2\cdot y2(x);$$

$$y := x \rightarrow y0(x) + \varepsilon\, y1(x) + \varepsilon^2\, y2(x)$$

We substitute the solution into the right-hand side of the system equation and then into the left-hand side:

$$eq2 := 4\cdot\varepsilon\cdot x - \left(y0(x) + \varepsilon\cdot y1(x) + \varepsilon^2\cdot y2(x)\right)^2;$$
$$eq3 := diff(y0(x),x) + \varepsilon\cdot diff(y1(x),x) + \varepsilon^2\cdot diff(y2(x),x) = 4\cdot\varepsilon\cdot x - \left(y0(x) + \varepsilon\cdot y1(x) + \varepsilon^2\cdot y2(x)\right)^2;$$

$$eq2 := 4\,\varepsilon\, x - \left(y0(x) + \varepsilon\, y1(x) + \varepsilon^2\, y2(x)\right)^2$$
$$eq3 := \frac{d}{dx}\, y0(x) + \varepsilon\left(\frac{d}{dx}\, y1(x)\right) + \varepsilon^2\left(\frac{d}{dx}\, y2(x)\right) = 4\,\varepsilon\, x - \left(y0(x) + \varepsilon\, y1(x) + \varepsilon^2\, y2(x)\right)^2$$

Transform the expression $eq3$ so that the left part contains summands with parameter ε and the right part summands without it. This can be done using the method *isolate* [11].

$$eq4 := isolate(eq3, \varepsilon);$$
$$eq4 := y2(x)^2\,\varepsilon^4 + 2\,y1(x)\,y2(x)\,\varepsilon^3 + 2\,y0(x)\,y2(x)\,\varepsilon^2 + y1(x)^2\,\varepsilon^2 + 2\,y0(x)\,y1(x)\,\varepsilon + \varepsilon^2\left(\frac{d}{dx}\, y2(x)\right) - 4\,\varepsilon\, x + \varepsilon\left(\frac{d}{dx}\, y1(x)\right) = -\left(\frac{d}{dx}\, y0(x)\right) - y0(x)^2$$

The terms of the expression need to be raised to the power of a small parameter to continue the solution. In order to do this, we will use the command *collect*, specifying in the syntax the parameter of which we need the degree to represent the expression[11]:

$$eq5 := collect(eq4, \varepsilon);$$
$$eq5 := y2(x)^2\,\varepsilon^4 + 2\,y1(x)\,y2(x)\,\varepsilon^3 + \left(2\,y0(x)\,y2(x) + y1(x)^2 + \frac{d}{dx}\, y2(x)\right)\varepsilon^2 + \left(2\,y0(x)\,y1(x) - 4\,x + \frac{d}{dx}\, y1(x)\right)\varepsilon = -\left(\frac{d}{dx}\, y0(x)\right) - y0(x)^2$$

In the latter case, the right-hand side is the coefficient at $\varepsilon = 0$, or otherwise represents the right-hand side of the system equation at $\varepsilon = 0$. Let's define the equation to find $y0(x)$:

$$eq\_0 := rhs(eq5) = 0;$$
$$eq\_0 := -\left(\frac{d}{dx}\, y0(x)\right) - y0(x)^2 = 0$$

There is an expression for finding $y1(x)$, which must be equal to zero, in the left part of the coefficient at ε^1.

$$eq\_1 := coeff(lhs(eq5), \varepsilon) = 0;$$
$$eq\_1 := 2\,y0(x)\,y1(x) - 4\,x + \frac{d}{dx}\, y1(x) = 0$$

Similarly, the coefficient at ε^2 is the left side of the equation for finding the $y2(x)$:

$$eq\_2 := coeff(lhs(eq5), \varepsilon^2) = 0;$$
$$eq\_2 := 2\,y0(x)\,y2(x) + y1(x)^2 + \frac{d}{dx}\, y2(x) = 0\,.$$

The initial conditions also have a representation in the form of a series of powers ε :

$$rNUS := y0(1) + \varepsilon \cdot y1(1) + \varepsilon^2 \cdot y2(1) = 1;$$

$$rNUS := y0(1) + \varepsilon\, y1(1) + \varepsilon^2\, y2(1) = 1$$

Let us write out the coefficients on powers of the ε expression of the series with initial conditions. For the coefficient at ε^0, we use the $subs()$ command to substitute $\varepsilon = 0$. The coefficients at the first and second degree ε, we will use $coeff()$:

$$k00 := subs(\varepsilon = 0, lhs(rNUS)) = subs(\varepsilon = 0, rhs(rNUS));$$

$$k11 := coeff(lhs(rNUS), \varepsilon) = coeff(rhs(rNUS), \varepsilon);$$
$$k22 := coeff(lhs(rNUS), \varepsilon^2) = coeff(rhs(rNUS), \varepsilon^2);$$

$$k00 := y0(1) = 1$$
$$k11 := y1(1) = 0$$
$$k22 := y2(1) = 0$$

At analytical solution of the problem we came to the same expressions of initial conditions:
$y_0(1) = 1,\ y_1(1) = 0,\ y_2(1) = 0$.

Find $y_0(x)$, using the command $desolve()$, in the command syntax specifying equation eq_0 and its corresponding initial condition

$$dsolve(\{eq\_0, lhs(k00) = rhs(k00)\}, y0(x));\ assign(\%);\ y0(x);$$

$$y0(x) = \frac{1}{x}$$

$$\frac{1}{x}$$

Analogously, we do the same for $y_1(x)$:

$$dsolve(\{eq\_1, lhs(k11) = rhs(k11)\}, y1(x));\ assign(expand(\%));\ y1(x);$$

$$y1(x) = \frac{x^4 - 1}{x^2} \quad x^2 - \frac{1}{x^2}$$

The solution for $y_2(x)$ is:

$$dsolve(\{eq\_2, lhs(k22) = rhs(k22)\}, y2(x)); assign(\%);\ y2(x);$$

$$y2(x) = -\frac{1}{7}x^5 + \frac{2}{3}x + \frac{1}{x^3} - \frac{32}{21x^2} \quad -\frac{1}{7}x^5 + \frac{2}{3}x + \frac{1}{x^3} - \frac{32}{21x^2}$$

To find

$$Y := eval(y(x));$$

$$Y := \frac{1}{x} + \varepsilon\left(x^2 - \frac{1}{x^2}\right) + \varepsilon^2\left(-\frac{1}{7}x^5 + \frac{2}{3}x + \frac{1}{x^3} - \frac{32}{21x^2}\right)$$

Let's return to the series with initial conditions:

$$rNUS := y0(1) + \varepsilon \cdot y1(1) + \varepsilon^2 \cdot y2(1) = 1;$$

The process of finding each of the initial conditions of the differential equation is carried out automatically. So let us change the initial condition $y(1) = 1$ to $y(1) = \varepsilon$. We have:

$rNUS := y0(1) + \varepsilon \cdot y1(1) + \varepsilon^2 \cdot y2(1) = \varepsilon;$
$k00 := subs(\varepsilon = 0, lhs(rNUS)) = subs(\varepsilon = 0, rhs(rNUS));$

$k11 := coeff(lhs(rNUS), \varepsilon) = coeff(rhs(rNUS), \varepsilon);$
$k22 := coeff(lhs(rNUS), \varepsilon^2) = coeff(rhs(rNUS), \varepsilon^2);$

$$rNUS := y0(1) + \varepsilon\, y1(1) + \varepsilon^2\, y2(1) = \varepsilon$$
$$k00 := y0(1) = 0$$
$$k11 := y1(1) = 1$$
$$k22 := y2(1) = 0$$

Due to the change, we have the following solution:

$dsolve(\{eq\_0, lhs(k00) = rhs(k00)\}, y0(x));$
$assign(\%);$
$y0(x);$
$dsolve(\{eq\_1, lhs(k11) = rhs(k11)\}, y1(x));$
$assign(expand(\%));$
$y1(x);$
$dsolve(\{eq\_2, lhs(k22) = rhs(k22)\}, y2(x));$
$assign(\%);$
$y2(x);$
$Y := eval(y(x));$

$$y0(x) = 0$$
$$0$$
$$y1(x) = 2x^2 - 1$$
$$2x^2 - 1$$
$$y2(x) = -\frac{4}{5}x^5 + \frac{4}{3}x^3 - x + \frac{7}{15}$$
$$-\frac{4}{5}x^5 + \frac{4}{3}x^3 - x + \frac{7}{15}$$
$$Y := \varepsilon\,(2x^2 - 1) + \varepsilon^2\left(-\frac{4}{5}x^5 + \frac{4}{3}x^3 - x + \frac{7}{15}\right)$$

It is necessary to first decompose the function into a Taylor series if the initial conditions are written in the form of a function with a small argument. For example, let the initial condition have the form $y(1) = \sin(\varepsilon)$. The function $f(x) = \sin x$ in Maple will have this representation:

$f := x \to \sin(x);$
$RT := taylor(f(x), x = 0, 6);$
$PL6 := convert(taylor(f(x), x = 0, 6), polynom);$

$$f := x \to \sin(x)$$
$$RT := x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + O(x^7)$$
$$PL6 := x - \frac{1}{6}x^3 + \frac{1}{120}x^5$$

Let's use the decomposition for $\sin\varepsilon$ last entry [11]:

$convert(taylor(sin(\varepsilon), x = 0, 6), polynom);$

$$\sin(\varepsilon)$$

As we can see, Maple did not perform the Taylor series expansion. Let us write as follows:

$$subs(x = \varepsilon, convert(taylor(f(x), x = 0, 6), polynom));$$

$$\varepsilon - \frac{1}{6}\varepsilon^3 + \frac{1}{120}\varepsilon^5$$

The Taylor series expansion is obtained. In accordance with the result, we introduce new initial data into the series with initial conditions:

$$rNUS := y0(1) + \varepsilon \cdot y1(1) + \varepsilon^2 \cdot y2(1) = subs(x = \varepsilon, convert(taylor(f(x), x = 0, 6), polynom));$$
$$k00 := subs(\varepsilon = 0, lhs(rNUS)) = subs(\varepsilon = 0, rhs(rNUS));$$
$$k11 := coeff(lhs(rNUS), \varepsilon) = coeff(rhs(rNUS), \varepsilon);$$
$$k22 := coeff(lhs(rNUS), \varepsilon^2) = coeff(rhs(rNUS), \varepsilon^2);$$

$$rNUS := y0(1) + \varepsilon\, y1(1) + \varepsilon^2\, y2(1) = \varepsilon - \frac{1}{6}\varepsilon^3 + \frac{1}{120}\varepsilon^5$$
$$k00 := y0(1) = 0$$
$$k11 := y1(1) = 1$$
$$k22 := y2(1) = 0$$

If this happens, the decision is made according to the previously prescribed procedure without changing:

```
dsolve({eq_0, lhs(k00) = rhs(k00)}, y0(x));
assign(%);
y0(x);
dsolve({eq_1, lhs(k11) = rhs(k11)}, y1(x));
assign(expand(%));
y1(x);
dsolve({eq_2, lhs(k22) = rhs(k22)}, y2(x));
assign(%);
y2(x);
Y := eval(y(x));
```

$$y0(x) = 0$$
$$0$$
$$y1(x) = 2x^2 - 1$$
$$2x^2 - 1$$
$$y2(x) = -\frac{4}{5}x^5 + \frac{4}{3}x^3 - x + \frac{7}{15}$$
$$-\frac{4}{5}x^5 + \frac{4}{3}x^3 - x + \frac{7}{15}$$
$$Y := \varepsilon\left(2x^2 - 1\right) + \varepsilon^2\left(-\frac{4}{5}x^5 + \frac{4}{3}x^3 - x + \frac{7}{15}\right).$$

The code for the solution of a differential equation thus takes into account the initial condition that is dependent on a small parameter in the form of the argument of the function. The solution requires the decomposition of the function into a Taylor series after having written the function with the argument x and then substituting the argument $x = \varepsilon$ with the command $subs()$. Since the solution depends on the analytical representation of the unknown function, the code developed to solve the Cauchy problem (1) by the small-parameter method is not automated. Nevertheless, the solution algorithm can be regarded as a technique for solving the differential equation by the small parameter method. It is undeniable that the solution in Maple is efficient in terms of calculation speed and the amount of work involved in the calculation process.

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