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RANDOM NONLINEAR VIBRATIONS OF AN IMPERFECTLY ELASTIC CIRCULAR PLATE WITH A DYNAMIC ABSORBER

Abstract: *This work considers the problem of random nonlinear transverse vibrations of an imperfectly elastic circular plate in combination with a dynamic absorber. Single-valued nonlinear functions representing the hysteresis-type absorber properties of the materials of the plate and the elastic damping element of the dynamic absorber were replaced with linear complex functions using the statistical linearization method and taken into account in the equations. A methodology for solving the problem has been developed. In order to investigate the dynamics and stability of the vibration-protected plate at different spectral densities of random processes, expressions for the root mean square values of the amplitudes and absolute accelerations were determined.*

Key words: *imperfectly elastic, circular plate, dynamic absorber, random vibrations, dissipative, hysteresis, root mean square value.*
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Introduction

Dynamic dampers, shock absorbers, and dampers are widely used in all areas of technology to reduce harmful vibrations, increase durability, and ensure long-term, high-quality, noise-free operation of machines and mechanisms, devices, and their elements.

Objects protected from vibrations, i.e. mechanical systems, are mathematically modeled in the form of solids, rods, plates, and shells.

The work [1] studies the dynamics of transverse vibrations of a circular elastic plate in combination with a dynamic damper. The finite element method is used to obtain the equation of motion of a circular plate under the influence of exponential impulse forces. The effects of transverse displacement deformation and rotational inertia are taken into account. The numerically analyzed results show that the vibration amplitude is rapidly attenuated by the action of the damper. It is shown that the pulse duration is one of the most important parameters in reducing amplitudes. It is analyzed that any small change in the pulse duration leads to a change in the vibration amplitude, and conclusions are drawn.

In the works [2-3], damped vibrations of a circular plate supported on an elastic foundation were studied based on classical plate theory. The differential equation of motion was derived and its solutions were determined. The frequencies, torsional functions and moments corresponding to the first two forms of vibrations for a circular plate were determined for the dynamic damper parameters and the elastic foundation under different values of the clamped and freely supported boundary conditions.

Recommendations were developed based on the results obtained.

In the work [4], an analytical model is proposed to study the vibration of a circular plate under impulse. The damping coefficient and velocity are simplified by introducing the damping force. Bending deformations are included in both the equilibrium equation and the yield state equation. Based on numerical analysis and analytical solution for rectangular impulse loading, the effect of the damping force on the vibration of a circular plate and around the resonance state is studied.

The work [5] shows that the transverse vibrations of each point of a circular plate with a constant thickness under the influence of stationary wide-band random excitations are also a wide-band random process. The root mean square values are determined and their changes are numerically analyzed. In this case, analytical and experimental analyses were carried out for an aluminum plate on a free support with a thickness of 0.16 cm and a diameter of 0.91 m, and conclusions were drawn on reducing the root mean square values. Analytical solutions were analyzed for excitations from 500 to 5000 Gs.

In the work [6], the vibrations of a circular plate subject to Kirchhoff's theory under the influence of random excitations were mathematically modeled. In this case, the random excitations were white noise, including a distributed load and a concentrated force. Two types of damper mechanisms, namely internal and external viscous dampers, were simultaneously taken into account in damping the vibrations. When solving boundary problems, the boundary conditions were fixed, freely supported, and mixed conditions. The eigenfrequencies were determined using the separation of variables method. The theory of mean square calculation was used to obtain analytical solutions for stationary random vibrations of a circular plate using the Bessel function. In this case, the functions representing the mean square values include the autocorrelation function and the spectral density function. The effect of parameters on the change in vibrations was evaluated based on numerical analyses.

In the works [7-11] the problems of nonlinear vibrations of a plate with dynamic absorbers and with dissipative characteristics of the hysteresis type and liquid links under various impact processes are considered.

MATERIALS AND METHODS

In this paper, we consider the problem of nonlinear transverse vibrations of an imperfectly elastic circular plate with a dynamic absorber under random influences.

The energy dissipation in the materials of the circular plate and the elastic damping element of the absorber is represented in the form of hysteresis friction due to imperfect elasticity, characterized by an ambiguous relationship between stresses and deformations. The analysis was performed using the method of statistical linearization in complex form [12]

$$\varepsilon V(\xi) = (-\bar{\eta}_1 + j\bar{\eta}_2)\xi\,, \tag{1}$$

where ε is small parameter; $\bar{\eta}_1, \bar{\eta}_2 = \bar{\eta}_{20} sign\omega$ are statistical linearization coefficients; ξ is relative deformation; ω is frequency.

The statistical linearization coefficients can be represented as

$$\begin{gathered}\bar{\eta}_{1r} = \eta_1 f_r(z);\ \bar{\eta}_{2r} = \eta_2 f_r(z);\\ \bar{\eta}_{1\theta} = \eta_1 f_\theta(z);\ \bar{\eta}_{2\theta} = \eta_2 f_\theta(z);\\ \bar{\nu}_{1r\theta} = \nu_1 g_\theta(z);\ \bar{\nu}_{2r\theta} = \nu_2 g_\theta(z),\end{gathered} \tag{2}$$

where

η_1, η_2, ν_1, ν_2 are constant coefficients depending on the dissipative properties of the plate material, determined from the corresponding dependence of the hysteresis loop; $f_r(z)$, $f_\theta(z)$ are vibration decrements as a function of maximum (amplitude) values of relative deformation; $g(z)$ is vibration decrement depending on the amplitude of relative shear deformation z;

$$f_r(z) = \sum_{i_1=0}^{k_1} C_{i_1} |V_1|^{i_1}_{\sigma_\xi} |z|^{i_1}; \qquad f_\theta(z) = \sum_{i_1=0}^{k_1} C_{i_1} |V_2|^{i_1}_{\sigma_\xi} |z|^{i_1};$$

$$g(z) = \sum_{i_2=0}^{k_2} K_{i_2} |V_3|^{i_2}_{\sigma_\xi} |z|^{i_2},$$

where

C_{i_1} $(i_1 = \overline{0, k_1})$, K_{i_2} $(i_2 = \overline{0, k_2})$ are some numbers (parameters) determined by points selected on experimental curves $\alpha_1 = f_r(z)$, $\alpha_2 = f_\theta(z)$, $\alpha_3 = g(z)$ with coordinates α_{1i}, α_{2i}, α_{3i} and z_i, obtained with a simple form of cyclic deformation of the material [13].

$$V_1 = \frac{\partial^2 w}{\partial r^2} + \mu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right);$$

$$V_2 = \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \mu\frac{\partial^2 w}{\partial r^2}; \qquad V_3 = \frac{1}{r}\frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta};$$

σ_ξ is root mean square value of relative plate deformation; w is deflection function in the direction perpendicular to the midplane of the plate; μ is Poisson's ratio; r, θ are polar coordinates.

Let us assume that axis x coincides with radius r, i.e. we take an angle of $\theta = 0$ between the abscissa axis and the polar axis. Then the bending moments M_r M_θ and the torque moment $M_{r\theta}$ acting in the cross section of the circular plate will have the form:

$$M_r = -DV_1[1 + 3(-\eta_1 + j\eta_2)\sum_{i_1=0}^{k_1} C_{i_1} |V_1|^{i_1}_{\sigma_\xi} \frac{h^{i_1}}{2^{i_1}(i_1+3)}]; \tag{3}$$

$$M_\theta = -DV_2[1+3(-\eta_1+j\eta_2)\sum_{i_1=0}^{k_1} C_{i_1}\left|V_2\right|^{i_1}_{\sigma_\xi}\frac{h^{i_1}}{2^{i_1}(i_1+3)}]; \qquad (4)$$

$$M_{r\theta} = -DV_3(1-\mu)[1+3(-\nu_1+j\nu_2)\sum_{i_2=0}^{k_2} K_{i_2}\left|V_3\right|^{i_2}_{\sigma_\xi}\frac{h^{i_2}}{2^{i_2}(i_2+3)}], \qquad (5)$$

where

$D=\dfrac{Eh^3}{12(1-\mu^2)}$ is cylindrical rigidity of the plate;

E is Young's modulus; h is plate thickness.

The derivation of differential equations for transverse vibrations of a circular plate with a absorber with dissipative characteristics of the hysteresis type under stationary random kinematic excitation is similar to the derivation of equations of motion of a system under harmonic excitation with the difference that instead of harmonic linearization coefficients, statistical linearization coefficients should be considered.

The system of differential equations of transverse nonlinear vibrations of a circular plate with a dynamic absorber, based on expressions (1) - (5), after transformations, we write in the form.

$$\begin{gathered}
D\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)^2 w+3(-\eta_1+j\eta_2)\cdot \\
\cdot\sum_{i_1=0}^{k_1} C_{i_1}\frac{h^{i_1}}{2^{i_1}(i_1+3)}\left[\frac{\partial^2}{\partial r^2}\left(V_1\left|V_1\right|^{i_1}_{\sigma_{\xi_o}}\right)+\right. \\
\left.+\left(\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\left(V_2\left|V_2\right|^{i_1}_{\sigma_{\xi_o}}\right)\right]+6D(1-\eta)(-\nu_1+j\nu_2)\cdot \\
\cdot\sum_{i_2=0}^{k_2} K_{i_2}\cdot\frac{h^{i_2}}{2^{i_2}(i_2+3)}\cdot\left(\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial}{\partial\theta}\right)\left(V_3\left|V_3\right|^{i_2}_{\sigma_{\xi_o}}\right)+ \\
+\rho h\frac{\partial^2 w}{\partial t^2}-cN\zeta\delta(r-r_0)\delta(\theta-\theta_0)=-\rho h\frac{\partial^2 w_0}{\partial t^2}; \\
\frac{\partial^2 w(r_0,\theta_0)}{\partial t^2}+\frac{\partial^2\zeta}{\partial t^2}+n^2N\zeta=-\frac{\partial^2 w_0}{\partial t^2},
\end{gathered} \qquad (6)$$

where $n=\sqrt{\dfrac{c}{m}}$ is natural frequency of a dynamic absorber; c, m are the coefficient of stiffness and the mass of the absorber, respectively; $\delta(r)$, $\delta(\theta)$ are Dirac delta functions; r_0, θ_0 are absorber installation coordinates; w_0 is base displacement; ζ is displacement of the absorber relative to the plate;

$$N=1+(-\theta_1+j\theta_2)[D_0+f(\sigma_{\zeta_o})];$$

where

θ_1, θ_2 are constant coefficients depending on the dissipative properties of the material of the elastic damping element of the absorber, determined from the corresponding dependence of the hysteresis loop;

$f(\sigma_{\zeta_0})$ is decrement of vibrations, represented in general form

$$f(\sigma_{\zeta_o})=D_1\sigma_{\zeta_o}+D_2\sigma^2_{\zeta_o}+\ldots+D_{s_1}\sigma^{s_1}_{\zeta_o};$$

σ_{ζ_0} is root mean square value of relative deformation of the elastic damping element of the absorber; $D_0, D_1, \ldots, D_{s_1}$ are some numbers (parameters) determined by points selected on the experimental curve $\alpha_0=f(\sigma_{\zeta_0})$ with coordinates f_i and и $(\sigma_{\zeta_0})_i$, obtained with a simple form of cyclic deformation of the material.

We will seek solutions to the system of differential equations (6) in the form of a series expansion in terms of the forms of the plate's natural linear vibrations, limiting ourselves to the first form of vibrations; after transformations, we obtain the following system of differential equations:

$$\begin{gathered}
D\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)^2 PQT+3(-\eta_1+j\eta_2)\cdot \\
\cdot\sum_{i_1=0}^{k_1} C_{i_1}\frac{h^{i_1}}{2^{i_1}(i_1+3)}T\left|T\right|^{i_1}_a\cdot\left[\frac{\partial^2}{\partial r^2}\left(\alpha_1\left|\alpha_1\right|^{i_1}_a\right)+\right. \\
\left.+\left(\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\left(\alpha_2\left|\alpha_2\right|^{i_1}_a\right)\right]+6D(1-\eta)(-\nu_1+j\nu_2)\cdot \\
\cdot\sum_{i_2=0}^{k_2} K_{i_2}\frac{h^{i_2}}{2^{i_2}(i_2+3)}T\left|T\right|^{i_2}_a\cdot\left[\left(\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial}{\partial\theta}\right)\left(\alpha_3\left|\alpha_3\right|^{i_2}_a\right)\right]+ \\
+\rho hPQ\ddot{T}-cN\zeta\delta(r-r_0)\delta(\theta-\theta_0)=-\rho hW_0; \\
P_0Q_0\ddot{T}+\ddot{\zeta}+n^2N\zeta=-W_0,
\end{gathered} \qquad (7)$$

where

$P=P(r)$, $Q=Q(\theta)$ are some functions depending on the radius r and angle θ respectively, determined from the boundary conditions; $T=T(t)$ - function of time;

$$P_0=P(r_0);\quad Q_0=Q(\theta_0);$$

$$\alpha_1=Q\frac{\partial^2 P}{\partial r^2}+\mu\left(\frac{1}{r}Q\frac{\partial P}{\partial r}+\frac{1}{r^2}P\frac{\partial^2 Q}{\partial\theta^2}\right);$$

$$\alpha_3=\frac{1}{r}\frac{\partial P}{\partial r}\frac{\partial Q}{\partial\theta}-\frac{1}{r^2}P\frac{\partial Q}{\partial\theta};$$

$$\alpha_2=\frac{1}{r}Q\frac{\partial P}{\partial r}+\frac{1}{r^2}P\frac{\partial^2 Q}{\partial\theta^2}+\mu Q\frac{\partial^2 P}{\partial r^2};$$

$W_0=\dfrac{\partial^2 w_0}{\partial t^2}$ - acceleration of the base.

Using the Bubnov-Galerkin method, based on the orthogonality condition, after transformations for the one-term approximation we have:

$$\begin{gathered}
\ddot{T}_k+AT_k-d_3P_0Q_0N\zeta=-d_*W_0; \\
P_0Q_0\ddot{T}_k+\ddot{\zeta}+n^2N\zeta=-W_0,
\end{gathered} \qquad (8)$$

where

$$A=[1+(-\eta_1+j\eta_2)N_1+(-\nu_1+j\nu_2)N_2]\omega^2_{01};$$

$$N_1=\frac{3D}{\omega^2_{01}\rho hd_1}\sum_{i_1=0}^{k_1} C_{i_1}\frac{h^{i_1}}{2^{i_1}(i_1+3)}\left|T\right|^{i_1}_a G_{i_1};$$

$$G_{i_1}=\iint_s PQ[\frac{\partial^2}{\partial r^2}(\alpha_1|\alpha_1|_a^{i_1})++(\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2})(\alpha_2|\alpha_2|_a^{i_1})]ds;$$

$$N_2=\frac{6D(1-\mu)}{\omega_{01}^2\rho hd_1}\sum_{i_2=0}^{k_2}K_{i_2}\frac{h^{i_2}}{2^{i_2}(i_2+3)}|T|_a^{i_2}H_{i_2};$$

$$H_{i_2}=\iint_s PQ(\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial}{\partial\theta})(\alpha_3|\alpha_3|_a^{i_2})ds;$$

$$d_*=\frac{d_2}{d_1};\quad d_1=\int_0^{2\pi}Q^2d\theta\int_{r_0}^{R}P^2dr;$$

$$d_2=\int_0^{2\pi}Qd\theta\int_{r_0}^{R}Pdr;\quad d_3=\frac{c}{\rho hd_1}.$$

Solving the system of algebraic equations (8) and assuming $s=j\omega$, we write expressions for determining the mean square values of variables T_k and ζ $(k=\overline{1,n})$ in the form

$$\sigma_{T_k}^2=\int_{-\infty}^{\infty}\frac{(\omega^2-n^2N)d_*-d_3P_0Q_0N}{\omega^4-B\omega^2+n^2AN}S_{W_0}(\omega)d\omega; \qquad (9)$$

$$\sigma_{\zeta}^2=\int_{-\infty}^{\infty}\frac{(1-d_*P_0Q_0)\omega^2-A}{\omega^4-B\omega^2+n^2AN}S_{W_0}(\omega)d\omega, \qquad (10)$$

where

$B=(n^2+d_3P_0^2Q_0^2)N+A$; $S_{W_0}(\omega)$ is spectral density of vibration acceleration of the base.

As a result, taking into account the ratio $\ddot{w}_a=\ddot{w}+W_0$ and using expression (9), we obtain the expression of interest to us for the mean square value of the absolute vibration of a round plate with a dynamic vibration absorber

$$\sigma_{W_a}^2=\int_{-\infty}^{\infty}(1-\omega^2P(r)Q(\theta)\cdot \qquad (11)$$
$$\cdot\frac{(\omega^2-n^2N)d_*-d_3P_0Q_0N}{\omega^4-B\omega^2+n^2AN})S_{W_0}(\omega)d\omega.$$

CONCLUSION.

Thus, the obtained integral expressions (9) – (11) make it possible to most fully investigate the dynamics and stability of a circular plate with a dynamic absorber, taking into account the dissipative properties of materials for different types of spectral density of random processes.

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