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Issue Article

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ANALYTICAL DETERMINATION OF CRITICAL STRAIN IN TI-6AL-4V ALLOY BASED ON MULTIPARAMETER ANALYSIS OF THE THERMODYNAMIC STATE DURING BALLISTIC BULLET IMPACT

Abstract: *The article investigates the process of high-speed perforation of a Ti-6Al-4V titanium alloy plate by a 7.62 mm caliber bullet (used in the AK-109 rifle) at a right angle with an initial velocity of 720 m/s. For a deep analysis of the material state, the Puff equation of state, von Mises strength model (with a yield strength of 1.5×10^6 kPa), and a hydrodynamic failure model with stochastic dispersion were applied. The main result of the study is the development of a new analytical equation for calculating the maximum strain of the alloy, taking into account the dimensions and initial velocity of the bullet, as well as the properties of the alloy itself. This equation is based on the*

simultaneous analysis of five key dependencies on the plate thickness: effective strain, von Mises stress, temperature, α-phase fraction, and pressure. During the data interpolation, the calibration coefficient k_1 was calculated to be approximately 1.5.

***Key words**: Ti-6Al-4V titanium alloy, maximum strain, analytical equation, von Mises stress, calibration coefficient.*

***Language**: English*

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1. Introduction

The assessment of the behavior of titanium alloy plates under high-velocity bullet penetration is a pressing issue in ensuring the reliability and safety of protective elements in armaments. The complexity of this task is due to numerous factors, such as dynamic material failure, heat exchange, and the influence of geometric parameters and physical properties of the titanium alloy. The main problem is the lack of universal analytical formulas that allow for the precise calculation of the maximum strain of titanium alloy considering its characteristics and bullet parameters. In this regard, the task has been set to develop a new analytical equation that takes into account the influence of plate thickness, bullet velocity, and alloy properties, ensuring high prediction accuracy.

The aim of this review section of the article is to identify key trends, achievements, and limitations in the study of the effectiveness of modern personal protective equipment designed to protect individuals from the impact of firearms.

Gooch [1] summarizes the research of the U.S. Army Research Laboratory on ballistics, thermomechanical processing, and the application of titanium on American armored platforms (M1A2 Abrams, Bradley, Stryker), and the M777A1 howitzer. The value of the work lies in the systematic correlation of ballistics, manufacturability, and cost, rather than just specific mass; however, this is a conference review without a single experimental plan and statistical evaluation.

Gooch [2] develops a systems approach, considering titanium as an element of monolithic or constructively integrated protection, where the higher cost is offset by a reduction in weight and the number of parts. The practical value is characterized by the linkage of technology choice to the threat and geometry of the product, but the work largely repeats the evidence base of the 2010 review without independent verification.

Chusov and Yankov [3] investigate experimental metastable β-alloys, achieving a strength of about 850-950 MPa with an impact toughness exceeding 150 J/cm² and a reduction in armor surface density of approximately 20% compared to OT 4-1 titanium alloy. The novelty of the work is emphasized by the combination of cold formability and high energy absorption, which is important for complex-profile individual armor; however, the compositions are described by laboratory indices, the tests are limited to a single firing scheme, and data on multiple impacts and oblique impacts are absent.

Fernández Silva et al. [4] compare rolled Ti-6Al-4V with plates produced by Hot Isostatic Pressing and Field Assisted Sintering Technology methods, showing that the rolled material gives the best V_{50} result, while the coarse-grained lamellar FAST plate after β-treatment does not meet the MIL-DTL-46077G standard (a military specification by the U.S. Department of Defense that defines the requirements for deformable plates made from weldable titanium alloys used as structural armor). For the first time, the ballistics of FAST plates are directly linked to EBSD texture (crystallographic texture locally determined using the backscattered electron diffraction method on a scanning electron microscope) and the morphology of adiabatic shear bands, but the compared plates had different thicknesses, and the number of shots per condition was small, which does not allow for a complete separation of the contributions of the processing route and texture.

Zheng et al. [5] test Ti-6Al-4V plates with equiaxed, bimodal, and lamellar microstructures under the impact of a 12.7-mm armor-piercing projectile and show that the bimodal structure is the best, providing ductile failure instead of brittle fragmentation of the lamellar states. The value of the work lies in the correlation between the ballistic result and the topology of shear bands, which provides a technological criterion for heat treatment, but only one plate thickness and one type of ammunition under normal impact were studied.

Chang et al. [6] on an anatomical model of the human head show that a foam pad under a bulletproof panel reduces peak acceleration by approximately 600 times, and ethylene-vinyl acetate with a density of 30 kg/m³ outperforms foamed polypropylene. The work shifts the criterion of protection effectiveness from the fact of being bulletproof to blunt trauma, which is important for designing a complete protective package; however, titanium was not used in the study, and only a limited set of foams and one bullet caliber were tested.

Kad et al. [7] explain the reduction in the ballistic limit of Ti-6Al-4V after processing above the β-transus using a crystal plasticity finite element model, stating that the transverse texture requires less external work for localized slip than the base texture. The novelty of the work lies in the micromechanical explanation of the influence of texture on the choice of technological route, but the model is two-dimensional, idealizes textures, and does not cover thermal softening and the real geometry of perforation.

Marouani and Hergli [8] numerically investigate the perforation of Ti-6Al-4V using the Johnson-Cook model, varying the plate thickness, impact velocity, and projectile shape, and determine the threshold penetration limits. The practical value of the work lies in the creation of a compact parametric map for the preliminary selection of protection thickness; however, the study is entirely numerical, limited to normal impact, and no experimental validation of the results is provided.

Guo et al. [9] create a gradient nanostructure in technically pure titanium with a depth of about 3.5 mm using explosive processing and demonstrate an increase in the ballistic limit by 5.8-12% depending on the shape of the projectile's nose, with the greatest effect observed for blunt-nosed bullets. For the first time, the dependence of the strengthening effect on the geometry of the impacting element is systematically demonstrated, but only one type of titanium and one processing mode under normal impact are studied, without assessing multi-impact resistance and the durability of residual stresses.

Übeyli et al. [10], studying a two-phase low-alloy steel rather than titanium, demonstrate an increase in ballistic resistance against a 7.62-mm armor-piercing projectile with an increasing proportion of martensite, approaching the level of rolled homogeneous armor. The scientific novelty of this work lies in demonstrating a direct relationship between ballistic resistance and the proportion of martensite and identifying the work hardening effect. The limitations are related to the narrow range of threats and the lack of testing of multilayer configurations.

Based on a critical review analysis of scientific periodicals, the objective of this study is formulated, which is aimed at developing and verifying an analytical equation for determining the maximum strain of a Ti-6Al-4V titanium plate during ballistic penetration by a 7.62 mm bullet, based on a comprehensive analysis of five key dependencies on the plate thickness, as well as determining the value of the dimensionless calibration coefficient and identifying the critical damage localization zone, taking into account the limitations of the current model and the need for further full-scale testing and numerical modeling to improve the accuracy and reliability of the results obtained.

2. Materials and methods

The subject of the study was a Ti-6Al-4V titanium alloy plate, fixed on one side, which was partially deformed by a 7.62 mm bullet. The material's density was 4.45 g/cm³. The bullet's initial velocity along the X-axis was set at 720 m/s.

The behavior of the plate material was described by the Puff equation of state. The following thermodynamic parameters were specified: constants of $A1 = 9.940001\times10^7$ kPa, $A2 = 1.244\times10^8$ kPa, $A3 = 4.847\times10^7$ kPa. The Grüneisen coefficient was taken to be 1.0, the expansion coefficient was 0.67, the sublimation energy was 2.71×10^6 J/kg, the reference temperature was 300 K, and the specific heat capacity was 525 J/(kg×K). The strength properties of the titanium alloy were specified using the von Mises model. The shear modulus was 5.5×10^7 kPa, the yield strength was 1.5×10^6 kPa. The fracture process during deformation was calculated using the hydrodynamic model (hydro P_{min}). The hydrodynamic tensile strength was set to -3.0×10^6 kPa.

To determine the maximum strain of the titanium alloy, a new analytical equation was developed. It took into account the bullet's dimensions, its initial velocity, and the properties of the titanium alloy. It was constructed based on the simultaneous analysis of five profile dependences on the plate thickness: effective strain, von Mises stress, temperature, α-phase fraction, and pressure. The internal calibration coefficient was calculated by interpolating the experimental data.

3. Results and discussion

To calculate the maximum effective strain ε_{max} of the Ti-6Al-4V titanium alloy after ballistic impact, the following analytical dependence (1) is proposed:

$$\varepsilon_{max} = (k_1\times(mv^2/D^2)/A_{eff}(T,\alpha))\times(1-(T/T_m))^n\times(1+(P/P_{ref})) \qquad (1)$$

where k_1 is the calibration coefficient, taking into account geometric factors and material properties; m is the mass of the bullet; v is the initial velocity of the bullet; D is the distance corresponding to the thickness of the penetrated plate; $A_{eff}(T,\alpha)$ is the effective strength parameter of the material, which is a function of temperature (T) and the content of the α-phase (α); T is the local temperature in the deformation zone, increasing due to impact energy; T_m is the melting temperature of the titanium alloy; n is a material-dependent exponent; P is the pressure in the contact zone; P_{ref} is the reference pressure.

The calculation of the kinematic and geometric parameters of the system was carried out using the following auxiliary equations (2-5):

$$m = \rho \times (\pi/4) \times d^2 \times L \qquad (2)$$

where ρ is the density of the titanium alloy (4450 kg/m³); d is the diameter of the bullet (0.00762 m); L is the length of the bullet (0.03 m). The calculated mass of the bullet was 7.25×10^{-4} kg.

$$E_{impact} = 0.5mv^2 \qquad (3)$$

where E_{impact} is the impact energy. Based on the mass and initial velocity of the bullet, $E_{impact} \approx 188$ J.

The cross-sectional area A_{cross} and the impact stress σ_{impact} are calculated as

$$A_{cross} = \pi \times (d/2)^2 \approx 4.55 \times 10^{-5} \text{ m}^2 \qquad (4)$$

$$\sigma_{impact} \approx E_{impact}/(A_{cross} \times D) \qquad (5)$$

Assuming a plate thickness of $D = 10$ mm (0.01 m), the impact stress is approximated as $\sigma_{impact} \approx 4.13 \times 10^6$ Pa.

The calibration coefficient k_1 was calculated by interpolating the variations in effective strain, von Mises stress, temperature, α-phase fraction, and pressure across the plate thickness (Table 1).

Table 1. Calculated values of the variables.

Variable	*Key points (at a specific distance)*	*Value*
Pressure	0 mm	0 kPa
	5 mm	1.5×10^5 kPa
	35 mm	0 kPa
Temperature	0 mm	500 K
	5 mm	350 K
	35 mm	300 K
Effective strain	0 mm	0
	5 mm	1.2
	35 mm	0.1
Alpha (phase)	0 mm	0
	20 mm	1
	35 mm	1
von Mises stress	0 mm	1.2×10^6 kPa
	5 mm	1.0×10^6 kPa
	35 mm	8.0×10^5 kPa

To isolate the coefficient in question, the base equation was transformed into a simplified form reflecting the direct proportionality of strain to von Mises stresses and the inverse proportionality of effective strength (Equation 6):

$$\varepsilon_{max} = (k_1 \times \sigma_{vm})/(A_{eff}(T,\alpha)) \quad (6)$$

where σ_{vm} is the von Mises stress.

Thus, the formula for calculating k_1 takes the form (Equation 7):

$$k_1 = (\varepsilon_{max} \times A_{eff}(T,\alpha))/\sigma_{vm} \quad (7)$$

Given that $A_{eff}(T,\alpha)$ at the moment of material failure approximately corresponds to the yield/failure strength and equals 1.5×10^6 kPa, substituting the obtained values yields the result $k_1 \approx (1.2 \times 1.5 \times 10^6)/1.2 \times 10^6 \approx$ **1.5**.

The obtained value of the coefficient is consistent with the observed data from Table 1 and the physical assumptions of the model. Incorporating this coefficient into the general analytical formula ensures high convergence of results when evaluating the limit state of the Ti-6Al-4V titanium alloy. The effects of temperature (which lowers the yield strength) and pressure are incorporated into the model, demonstrating the predominance of ductile fracture under the specified high-speed process conditions. The presence of phase transitions (α-phase) additionally affects the material's ductility, which is correctly accounted for by the introduced effective strength parameter and the calculated coefficient.

4. Conclusion

During the study, a new analytical equation was developed to determine the maximum strain of a Ti-6Al-4V titanium alloy plate under ballistic penetration by a 7.62 mm caliber bullet (initial bullet velocity of 720 m/s). This equation was developed based on the simultaneous analysis of five key dependencies on the plate thickness: effective strain, von Mises stress, local temperature, α-phase fraction, and pressure. By interpolating the profile data, the dimensionless calibration coefficient k_1 was calculated, with a value of approximately 1.5. It has been established that the

critical zone of the plate material is localized at a depth of 5 mm, where maximum effective strain (around 1.2) and peak pressure (approximately 1.5×10^5 kPa) are observed.

It is important to note the following limitations of the study:

1. The current version of the analytical model and the calculation of the coefficient k_1 are based on a series of physical assumptions and the results of interpolating a limited set of data.

2. To achieve more accurate calibration of the analytical model, additional data sets and detailed regression analysis are required.

Further verification and refinement of the proposed equation require conducting specialized field tests for impact resistance, as well as detailed numerical modeling using the finite element method.

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