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Issue **Article**

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REGRESSION ANALYSIS AND HARMONIC APPROXIMATION OF THE STRESS STATE OF KEVLAR AND TITANIUM PLATES UNDER THE IMPACT OF A 7.62 MM CALIBER BULLET

Abstract: *The article is dedicated to the numerical study and mathematical modeling of the distribution of circumferential (tangential) stresses in protective plates made of Kevlar composite (Kevlar/epoxy resin) and Ti-6Al-4V titanium alloy under ballistic impact (bullet caliber of 7.62 mm, bullet velocity varies in the range of 500-900 m/s). It has been established that in the isotropic titanium alloy, the stress distribution across the plate thickness is described by a smooth parabolic dependence, whereas in the orthotropic Kevlar composite, pronounced oscillations are observed, caused by wave effects. The practical novelty lies in the development of a differentiated mathematical*

apparatus. The use of Fourier series for predicting local tensile zones in a Kevlar composite and 4th-5th degree polynomials for estimating stresses in a titanium alloy under ballistic impact is proposed. The resulting regression models and identified patterns can be directly used in the design and optimization of layer thicknesses for hybrid personal and composite body armor, minimizing the risk of material delamination while maintaining minimal weight.

***Key words**: Kevlar composite, Ti-6Al-4V alloy, circumferential stress, tangential stress, plate.*

***Language**: English*

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1. Introduction

During a ballistic impact, the Kevlar composite and titanium alloy are subjected to high loads, causing complex stresses, including circumferential (tangential) components. The main issues are related to their resistance to delamination, interlaminar shear, and failure under the influence of these stresses, especially in the areas of stress concentrators at the edges and in the contact zone with the impacting element. An important aspect is the study of material behavior under high loading rates, as circumferential (tangential) stresses significantly influence the failure mechanism, promoting the initiation and propagation of cracks, which reduces the effectiveness of protection and the reliability of the structure.

A number of researchers, through practical experiments and finite element modeling of complex ballistic impact processes, are expanding the concept of deformation mechanisms in armor materials and proposing effective ways to increase their efficiency while maintaining strength.

Yahaya et al. [1] systematize the materials, protection levels, and testing of bulletproof vests and helmets according to the standards of NIJ (National Institute of Justice), HOSDB (Home Office Scientific Development Branch), VPAM (Vereinigung der Prüfstellen für Angriffshemmende Materialien), GOST (State Standard), STANAG (Standardization Agreement), and other systems. The practical contribution of the article is a unified framework of "threat – conditions – pass criteria." Nevertheless, the classes of different standards are not directly comparable, and the regulated tests do not cover all cases of multiple and edge impacts, aging, and combined effects.

Abtew et al. [2] associate the ballistic response of textiles and fiber composites with wave propagation, stretching and breaking of fibers, friction, delamination, and matrix damage. Mechanistic systematization is useful for selecting the weave, the number of layers, and the matrix. The limitations of the narrative review are the heterogeneity of the trials and the lack of a universal quantitative model, especially without normalization by surface density.

Kumar et al. [3] numerically demonstrated for Kevlar composite plates an increase in ballistic limit and energy absorption with relative thickness. At high speeds, the dependence of residual velocity becomes almost linear. A three-dimensional model with progressive damage is suitable for preliminary ranking of structures. However, the sensitivity to failure criteria and contact, as well as the lack of a discrete description of fibers, are the main limitations of the study.

Kumar et al. [4] combined the wave description and energy balance of the ceramic faceplate, composite backing, and projectile. The model identified a range of surface density where increasing the mass hardly raises the ballistic limit. Among the options studied, the best result was achieved with 99.9% Al_2O_3. The practical value of the research lies in finding the minimum mass instead of simply increasing the thickness. The limitations are related to the averaging of fragmentation, erosion, and material delamination, as well as the dependence of the output on the configuration.

Stopforth and Adali [5] tested Kevlar layers with densities of 160, 200, and 400 g/m² using 9-mm ammunition. The best option was the 200 g/m², and the stop rating was up to 21 layers. The authors note the benchmark for package layout and the demonstration of the role of architecture, not just mass. Portability is limited by a single type of threat, a small number of shots, and an incomplete reproduction of the real armor package.

Tham et al. [6] compared the experiment on the Kevlar 29/PVB-phenolic resin helmet (the classic technological basis for the production of the first generations of series army ballistic helmets) with AUTODYN-3D. The calculation closely reproduced the dent and provided a V_{50} for the plate of 610 m/s compared to 600 m/s in the test. The work demonstrates the value of model verification based on damage geometry, not just penetration. The research is limited by the borrowed properties of Kevlar 129 and a narrow experimental base; predictions for other threats are partially calculated.

Lim et al. [7] developed an economical FE model of Twaron fabric (finite element model of high-strength aramid fabric) with viscoelasticity and rate-dependent failure. The residual velocity and energy absorption were in good agreement with the experiment, especially above 400 m/s. The novelty of

the study lies in the consideration of the rheological response in the homogenized model. The absence of explicit inter-fiber friction, entanglement, and fiber stretching, as well as the need for revalidation for other tissues and packages, are significant limitations of the study.

Kay [8] determined the failure parameters of the Johnson-Cook model (an empirical mathematical model used to predict the onset of failure in ductile metals under large deformations, high strain rates, and temperatures) for Ti-6Al-4V and Al 2024-T3 and showed that calibration based on representative dynamic tests transfers better than that based on reduced ballistic limit tests. This is important for calculating the retention of aviation fragments. As limitations, the authors note the dependence of parameters on scale and loading path, and the incomplete consideration of localization, microstructure, and complex stress state.

Awerbuch and Bodner [9] presented the perforation of a metal plate as three stages: target compression, shear plug formation, and then the simultaneous motion of the plug and the projectile. The mechanistic model allows for a quick assessment of the residual velocity and force history. Limitations include the need for empirical geometric parameters and the focus on normal impact with shear spalling. Petaling failure, ricochet, and complex thermomechanical failure are described incompletely.

Khodke et al. [10] conducted FE screening of titanium and aluminum plates based on thickness and impact angle at 830 m/s. At 45°, the calculation showed a transition from perforation to ricochet. The "angle – thickness – outcome" map is useful for preliminary screen selection. However, the limited scenarios, idealized fixation, and lack of complete experimental validation do not allow considering 45° as a universal critical angle or using the results as certification.

Having analyzed the gaps in scientific works, we focus our research on identifying and modeling the patterns of circumferential (tangential) stresses in Kevlar composite and Ti-6Al-4V titanium alloy under ballistic impact, as well as assessing their practical value for the development of effective protective structures. Particular attention is paid to the analysis of stress topology, the identification of potential failure zones, and the creation of mathematical models that allow predicting the mechanical responses of materials under dynamic loads, which contributes to the optimization of armor plate design and enhances their resistance to ballistic impacts.

2. Materials and methods

The study focused on the process of a 7.62 mm caliber bullet penetrating at a 90° angle into plates rigidly fixed on one side, made from different materials. The initial velocity of the bullet along the X-axis varied, with a baseline value of 720 m/s.

The analysis of the stress state was conducted on two types of materials:

1. Orthotropic Kevlar composite with a density of 1.65 g/cm³, a matrix volume fraction of 60%, and a modulus of elasticity ratio of 20. The state of the material was described by a stiffness matrix with a polynomial volumetric response and failure based on stresses/strains, taking into account the residual shear stiffness fraction.

2. Isotropic two-phase Ti-6Al-4V titanium alloy with a density of 4.45 g/cm³. The material behavior was described by the Puff equation of state and the von Mises plasticity model (shear modulus 5.5×10^7 kPa, yield strength 1.5×10^6 kPa) with a hydrostatic failure model (P_{min}).

Data acquisition was performed by recording the stress tensor component $\sigma_{\theta\theta}$ (circumferential (tangential) stress) as a function of plate thickness (x) and impact velocity (v).

3. Results and discussion

Numerical values of the change in circumferential (tangential) stress of the Kevlar composite plate and the Ti-6Al-4V titanium alloy were recorded along the bullet's flight path through the entire thickness of the plates. 3D graphics demonstrating the influence of plate thickness and bullet velocity on the magnitude and nature of the change in circumferential (tangential) stress of the materials were generated in the Python. 3D surfaces of circumferential (tangential) stresses of materials from plate thickness and bullet velocity (parameter variation simulation) are presented in Fig. 1.

The analysis of three-dimensional response surfaces revealed fundamental differences in the distribution characteristics and relaxation mechanisms of circumferential (tangential) stresses $\sigma_{\theta\theta}$ in the Ti-6Al-4V titanium alloy and the Kevlar composite during ballistic impact (bullet velocity ranged from 500 to 900 m/s, bullet caliber is 7.62 mm).

The stress surface of the Ti-6Al-4V titanium alloy has a smooth, parabolic (U-shaped) form along the thickness coordinate x of the plate. The maximum compressive stresses are localized in the central zone of the plate, reaching equivalent values on the order of -25×10^5 kPa, after which they gradually decrease toward the rear and front boundaries. This profile is explained by the spherical symmetry of the shock wave front propagation and the kinematics of the plastic flow of the metal according to the von Mises criterion when the yield strength is exceeded (1.5×10^6 kPa). The collision velocity v acts as a nonlinear scaling factor: an increase in velocity amplifies the compression amplitude without altering the fundamental geometric shape of the spatial gradient.

The topology of stress fields in the Kevlar composite is characterized by pronounced spatial periodicity and oscillations. The values of $\sigma_{\theta\theta}$ cyclically change sign from compression (up to -

8×10^5 kPa) to tension (up to 2×10^5 kPa). The physical nature of this phenomenon lies in the high anisotropy of the elastic moduli (E_1/E_2 = 20) and the regular alternation of fiber and matrix layers. When a ballistic wave passes through interphase boundaries, multiple processes of reflection and refraction of elastic impulses occur, leading to their interference and the formation of standing stress waves across the thickness of the plate.

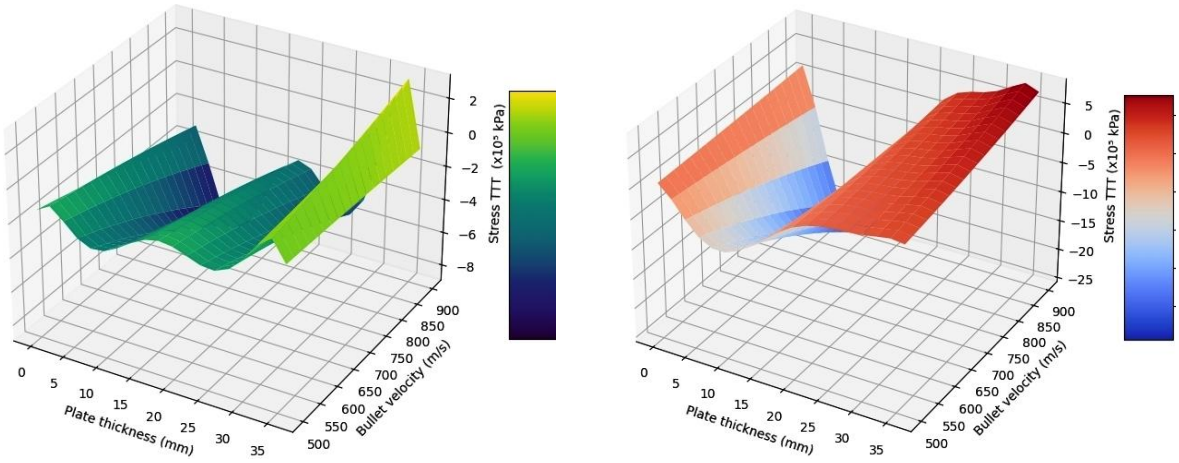


Figure 1. 3D response surface graphs of circumferential (tangential) stress changes as a function of plate thickness and bullet velocity: a – Kevlar composite and b – Ti-6Al-4V titanium alloy. Stress TTT is circumferential (tangential) stress.

The smooth nature of the decay of circumferential (tangential) stresses in the titanium alloy allows the use of a 4th or 5th degree polynomial (Equation 1):

$$\sigma_{\theta\theta}(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \ldots + \beta_n x^n \quad (1)$$

where x is the coordinate along the thickness of the plate; $\beta_0 \ldots \beta_n$ are the coefficients of the polynomial regression (calculated using the least squares method to fit the data); n is the degree of the polynomial.

This model provides high-quality approximation ($R^2 > 0.95$).

The application of power polynomials to the data of Kevlar composite leads to significant smoothing and loss of peak stress values. Therefore, to describe the state of the Kevlar composite under ballistic impact conditions, a Fourier series with a spatial wave number $k_n = \frac{2\pi n}{L}$ (Equation 2) is used:

$$\sigma_{\theta\theta}(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n cos\left(\frac{2\pi n x}{L}\right) + b_n sin\left(\frac{2\pi n x}{L}\right)\right) \quad (2)$$

where a_0 is the constant term (zero coefficient) of the Fourier series, determining the average value of the function; n is the harmonic number; a_n and b_n are the harmonic coefficients of the Fourier series of the n-th harmonic; L is the characteristic length or total thickness of the plate.

Equation (2) allows for an accurate description of the amplitude and phase of the oscillations. The lower harmonics define the macro-profile of the plate deformation, while the higher harmonics account for the stress jumps at the boundaries of individual material layers.

The practical value of the obtained patterns is as follows:

1. Zones where the circumferential (tangential) stress takes on positive values ($\sigma_{\theta\theta} > 0$) correspond to maximum tensile forces leading to delamination of the polymer matrix from the aramid fibers. Calculating the zeros and extrema of the harmonic Fourier function allows one to determine the x-coordinate of potential interlayer failure with high accuracy without performing labor-intensive calculations.

2. The identified difference in the topology of circumferential (tangential) stresses argues for the feasibility of creating combined structures: the front layer of the plate made of Ti-6Al-4V titanium alloy ensures the localization of plastic deformation and the damping of the main kinetic impulse, and the back layer of the plate made of Kevlar composite, due to wave dissipation, effectively distributes tangential loads and captures fragments of destruction.

4. Conclusion

The results of the study showed that the structural features of the materials under consideration

fundamentally determine the topology of stress fields during a ballistic bullet impact.

It is shown that Ti-6Al-4V titanium alloy generates a smooth stress gradient described by polynomial functions, while Kevlar composite exhibits oscillating elastic wave behavior. The application of harmonic analysis (Fourier series) for predictive modeling of phase transition zones from compression to tension in layered aramid plates is substantiated, which allows mathematical prediction of the depth of potential material delamination upon bullet penetration.

The study results have a number of limitations. Current regression models are limited to a range of initial bullet velocities of 500-900 m/s and a bullet-plate contact angle of exactly 90°. In addition, the macroscopic model does not take into account the microstructural destruction of individual fibers of the Kevlar composite at the point of direct contact with the bullet, which requires further micromechanical studies.

References:

1. Yahaya, R., Hidayah, N., Norhayaty, Z., Nor Hafizah, M. J., Sapuan, S. M., & Ilyas, R. A. (2021). Levels of Ballistic Protection and Testing. In: Sapuan, Y. N. S. M., & Shaker, K., Eds., *Composite Solutions for Ballistics. Elsevier*, 77-108.
2. Abtew, M. A., et al. (2019). Ballistic impact mechanisms – a review on textiles and fibre-reinforced composites impact responses. *Compos. Struct., 223*, 1-41.
3. Kumar, S., Gupta, D. S., Singh, I., & Sharma, A. (2010). Behavior of Kevlar/epoxy composite plates under ballistic impact. *Journal of Reinforced Plastics and Composites, 29*, 2048-64.
4. Kumar, S., et al. (2020). Performance of ceramic-composite armors under ballistic impact loading. *J. Materi. Eng. Perform., 29*, 5625-5637.
5. Stopforth, R., & Adali, S. (2019). Experimental Study of Bullet-Proofing Capabilities of Kevlar, of Different Weights and Number of Layers, with 9 mm Projectiles. *Defence Technology, 15*, 186-192.
6. Tham, C. Y., Tan, V. B. C., & Lee, H. P. (2008). Ballistic impact of a KEVLAR® helmet: experiment and simulations. *International Journal of Impact Engineering, 35 (5)*, 304-318.
7. Lim, C. T., Shim, V. P. W., & Ng, Y. H. (2003). Finite-element modeling of the ballistic impact of fabric armor. *International Journal of Impact Engineering, Vol. 28*, 13-31.
8. Kay, G. (2003). Failure modeling of titanium 6Al–4V and aluminum 2024–T3 with the Johnson-Cook material model. *Report No.: DOT/FAA/AR-03/57*.
9. Awerbuch, J., & Bodner, S. R. (1973). Analysis of the mechanics of perforation of projectiles in metallic plates. *International Journal of Solids and Structures, 10*, 671-684.
10. Khodke, M., et al. (2024). Finite Element Simulation on Ballistic Impact of Bullet on Metal Plate. In: Jha, P. K., Tripathi, B., Natarajan, E., Sharma, H. (eds). *Proceedings of Congress on Control, Robotics, and Mechatronics. CRM 2023, Smart Innovation, Systems and Technologies, vol. 364, Springer, Singapore*.